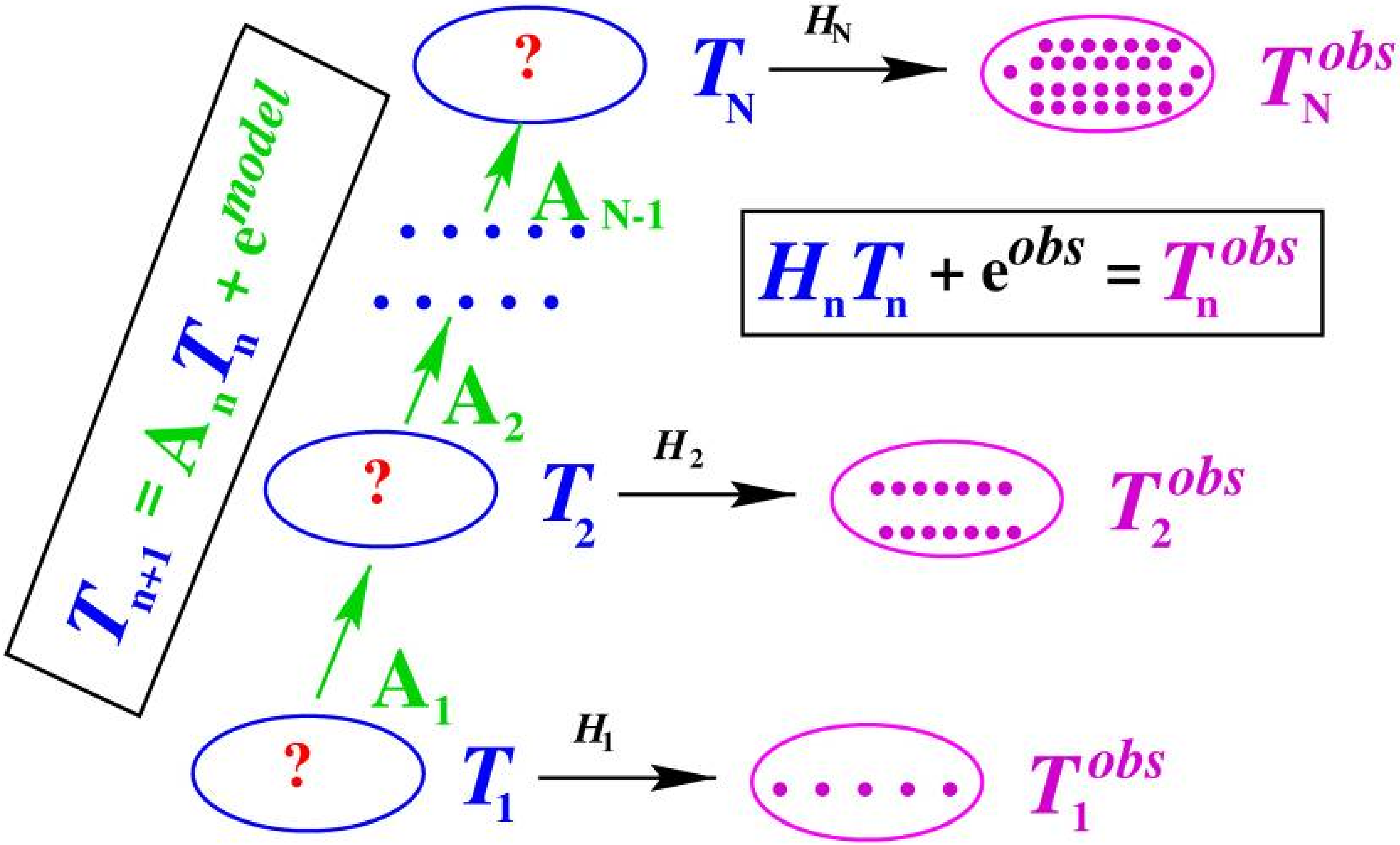
Reduced Space Data Assimilation and Historical Sea Surface Temperature Analyses

Alexey Kaplan

Lamont-Doherty Earth Observatory (LDEO) of Columbia University

Generic problem of the analysis of time-evolving fields



GENERAL PROBLEM OF ANALYSIS OF A HISTORICAL DATASET

$$\mathcal{T}_n^o = H_n \mathcal{T}_n + \varepsilon_n^o, \quad n = 1, \ldots, N.$$

$$\mathcal{T}_{n+1} = A_n \mathcal{T}_n + \varepsilon_n^{\mathrm{m}}, \quad n = 1, \ldots, N-1$$

$$\langle \varepsilon_n^o \rangle = 0, \qquad \langle \varepsilon_n^o \varepsilon_n^{o\,T} \rangle = R_n, \quad n = 1, \ldots, N$$

$$\langle \varepsilon_{n_1}^o \varepsilon_{n_2}^{o\,T} \rangle = 0, \quad n_1 \neq n_2, \quad \langle \varepsilon_{n_1}^o \mathcal{T}_{n_2}^T \rangle = 0, \quad n_1, n_2 = 1, \ldots, N,$$

$$\langle \varepsilon_n^{\mathrm{m}} \rangle = 0, \qquad \langle \varepsilon_n^{\mathrm{m}} \varepsilon_n^{\mathrm{m}\,T} \rangle = Q_n, \quad n = 1, \ldots, N-1$$

$$\langle \varepsilon_{n_1}^{\mathrm{m}} \varepsilon_{n_2}^{\mathrm{m}\,T} \rangle = 0, \quad n_1 \neq n_2, \quad \langle \varepsilon_{n_1}^{\mathrm{m}} \mathcal{T}_{n_2}^T \rangle = 0, \quad n_1, n_2 = 1, \ldots, N-1$$

$$\langle \varepsilon_{n_1}^o \varepsilon_{n_2}^{\mathrm{m}\,T} \rangle = 0, \quad n_1 = 1, \ldots, N, \qquad n_2 = 1, \ldots, N-1$$

Gauss–Markov Theorem

If $\mathcal{T}^o = H\mathcal{T} + \varepsilon$,
$\langle\varepsilon\rangle = 0, \quad \langle\varepsilon\varepsilon^T\rangle = R, \quad \langle\varepsilon\mathcal{T}^T\rangle = 0$,
then the Least Squares Estimate (LSE)

$$\hat{\mathcal{T}} = (H^T R^{-1} H)^{-1} H^T R^{-1} \mathcal{T}^o$$

minimizes

$$S[\mathcal{T}] = (H\mathcal{T} - \mathcal{T}^o)^T R^{-1} (H\mathcal{T} - \mathcal{T}^o)$$

and is the **Best**[1] **Linear Unbiased Estimate (BLUE)** with error covariance

$$P \stackrel{\text{def}}{=} \langle(\mathcal{T} - \hat{\mathcal{T}})(\mathcal{T} - \hat{\mathcal{T}})^T\rangle = (H^T R^{-1} H)^{-1}.$$

ε is normal $\Longrightarrow$ $\mathcal{T}$ is a **Maximum Likelihood Estimate (MLE)**
ε and $\mathcal{T}$ are normal $\Longrightarrow$ $\mathcal{T}$ is the best among **all** (not necessarily linear) estimates.

---

1 $\|\mathcal{T} - \hat{\mathcal{T}}\|_S^2 = \langle(\mathcal{T} - \hat{\mathcal{T}})^T S (\mathcal{T} - \hat{\mathcal{T}})\rangle \longrightarrow \min \quad \forall S \Rightarrow$ minimal variance

TRANSFER TO GAUSS–MARKOV SCHEME

$$\tilde{\mathcal{T}} = \begin{bmatrix} \mathcal{T}_1 \\ \mathcal{T}_2 \\ \mathcal{T}_3 \\ \vdots \\ \mathcal{T}_{N-1} \\ \mathcal{T}_N \end{bmatrix}, \quad \tilde{\mathcal{T}}^o = \begin{bmatrix} \mathcal{T}_1^o \\ \mathcal{T}_2^o \\ \mathcal{T}_3^o \\ \vdots \\ \mathcal{T}_{N-1}^o \\ \mathcal{T}_N^o \end{bmatrix}, \quad \tilde{\varepsilon}^o = \begin{bmatrix} \varepsilon_1^o \\ \varepsilon_2^o \\ \varepsilon_3^o \\ \vdots \\ \varepsilon_{N-1}^o \\ \varepsilon_N^o \end{bmatrix}, \quad \tilde{\varepsilon}^{\mathrm{m}} = \begin{bmatrix} \varepsilon_1^{\mathrm{m}} \\ \varepsilon_2^{\mathrm{m}} \\ \varepsilon_3^{\mathrm{m}} \\ \vdots \\ \varepsilon_{N-1}^{\mathrm{m}} \end{bmatrix}, \tilde{\mathcal{T}}^{\mathrm{m}} = \tilde{\varepsilon}^{\mathrm{m}} \cdot 0$$

$$\tilde{\mathcal{T}}^{o\,\mathrm{m}} = \tilde{H}\tilde{\mathcal{T}} + \tilde{\varepsilon}$$

$$\tilde{H} = \begin{bmatrix} \tilde{H}^o \\ \tilde{H}^{\mathrm{m}} \end{bmatrix}, \quad \mathcal{T}^{\tilde{o}\,\mathrm{m}} = \begin{bmatrix} \tilde{\mathcal{T}}^o \\ \tilde{\mathcal{T}}^{\mathrm{m}} \end{bmatrix}, \quad \tilde{\varepsilon} = \begin{bmatrix} \tilde{\varepsilon}^o \\ -\tilde{\varepsilon}^{\mathrm{m}} \end{bmatrix}$$

$$\tilde{H}^o = \begin{bmatrix} H_1 & & & & & \\ & H_2 & & & 0 & \\ & & H_3 & & & \\ & & & \ddots & & \\ & 0 & & & H_{N-1} & \\ & & & & & H_N \end{bmatrix}, \quad \tilde{H}^{\mathrm{m}} = \begin{bmatrix} -A_1 & I_M & & & & \\ & -A_2 & I_M & & 0 & \\ & & -A_3 & I_M & & \\ & 0 & & \ddots & \ddots & \\ & & & & -A_{N-1} & I_M \end{bmatrix}$$

$$< \tilde{\varepsilon} >= 0, \qquad < \tilde{\varepsilon}\tilde{\varepsilon}^T > \stackrel{\mathrm{def}}{=} \tilde{R} = \begin{bmatrix} R_1 & & & & & & & \\ & R_2 & & & & & & \\ & & \ddots & & & 0 & & \\ & & & R_N & & & & \\ & & & & Q_1 & & & \\ & & 0 & & & Q_2 & & \\ & & & & & & \ddots & \\ & & & & & & & Q_{N-1} \end{bmatrix}$$

$$\hat{\tilde{\mathcal{T}}} = \left(\tilde{H}^T \tilde{R}^{-1} \tilde{H}\right)^{-1} \tilde{H}^T \tilde{R}^{-1} \tilde{\mathcal{T}}^{o\,\mathrm{m}}$$

$$\tilde{\mathbf{S}}[\tilde{\mathcal{T}}] = \left(\tilde{H}\tilde{\mathcal{T}} - \tilde{\mathcal{T}}^{o\,\mathrm{m}}\right)^T \tilde{R}^{-1} \left(\tilde{H}\tilde{\mathcal{T}} - \tilde{\mathcal{T}}^{o\,\mathrm{m}}\right)$$

MINIMIZATION OF THE FULL COST FUNCTION:

OPTIMAL SMOOTHER (OS) and KALMAN FILTER (KF)

Cost function:

$$\mathbf{S}[\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_N] = \sum_{n=1}^{N} (H_n \mathcal{T}_n - \mathcal{T}_n^o)^T R_n^{-1} (H_n \mathcal{T}_n - \mathcal{T}_n^o) + \sum_{n=1}^{N-1} (\mathcal{T}_{n+1} - A_n \mathcal{T}_n)^T Q_n^{-1} (\mathcal{T}_{n+1} - A_n \mathcal{T}_n)$$

"Sweep up" – **KF**:

$$\hat{\mathcal{T}}_n^a = \hat{\mathcal{T}}_n^f + K_n \left(\hat{\mathcal{T}}_n^o - H_n \hat{\mathcal{T}}_n^f \right),$$

$$\hat{\mathcal{T}}_n^f = A_n \hat{\mathcal{T}}_{n-1}^a,$$

$$K_n = P_n^f H_n^T \left(H_n P_n^f H_n^T + R_n \right)^{-1}$$

$$P_n^a = (I_M - K_n H_n) P_n^f$$

$$P_n^f = A_{n-1} P_{n-1}^a A_{n-1}^T + Q_{n-1}, \qquad n = 2, 3, \ldots, N$$

"Sweep down" – **OS**:

$$\hat{\mathcal{T}}_n^s = \hat{\mathcal{T}}_n^a + G_n \left(\hat{\mathcal{T}}_{n+1}^s - A_n \hat{\mathcal{T}}_n^a \right), \qquad G_n = P_n^a A_n^T (P_{n+1}^f)^{-1},$$

$$P_n^s = P_n^a + G_n \left(P_{n+1}^s - P_{n+1}^f \right) G_n^T, \qquad n = N-1, \ldots, 2, 1$$

SIMPLIFIED CASE: OPTIMAL INTERPOLATION (OI)

Cost functions:

$$\mathbf{S}_n[\mathcal{T}_n] = (H_n\mathcal{T}_n - \mathcal{T}_n^o)^T R_n^{-1}(H_n\mathcal{T}_n - \mathcal{T}_n^o) + \mathcal{T}_n^T C_n^{-1}\mathcal{T}_n, \qquad n > 1.$$

OI solution:

$$\mathcal{T}_n^{\mathrm{OI}} = P_n^{\mathrm{OI}} H_n^T R_n^{-1}\mathcal{T}_n^o, \quad P_n^{\mathrm{OI}} = \left(H_n^T R_n^{-1} H_n + C_n^{-1}\right)^{-1}.$$

Example of Optimal Interpolation

$$T = T_B + e_B$$

$$HT = T_o + e_o$$

$$\langle e_B \rangle = \langle e_o \rangle = \langle e_B e_o^T \rangle = 0$$

$$\langle e_B e_B^T \rangle = C$$ ← **Hard to know in detail!**

$$\langle e_o e_o^T \rangle = R$$

Solution minimizes the cost function

$$S[T] = (HT - T_o)^T R^{-1} (HT - T_o) + (T - T_B)^T C^{-1} (T - T_B)$$

$$T = (H^T R^{-1} H + C^{-1})^{-1} (H^T R^{-1} T_o + C^{-1} T_B)$$

Projection of OI solution on eigenvectors of *C* (EOFs)

$C = EDE^T$

$T = Ea$

For simplicity: $H = I$, $R = rI$, $T := T - T_B$

Then $a = \underline{D(D+R)^{-1}} E^T T_o$

$D(D+R)^{-1} = \mathrm{diag}[\, d_i/(d_i+r)\,]$, $a_o = E^T T_o$

Therefore $a_i/a_o = d_i/(d_i+r)$

In many applications (for spectrally red signals) diagonal elements of this matrix decrease from ~1 to ~0. In effect, the solution is constrained to the subspace spanned by the patterns with $d_i >> r$.

SPACE REDUCTION

$$C = E\Lambda E^T + E'\Lambda' E'^T$$

$$\mathcal{T}_n = E\alpha_n + \varepsilon^r_n, \quad n = 1, \ldots, N$$

ESTIMATION PROBLEM IN THE REDUCED SPACE

$$\mathcal{T}^o_n = H_n E\alpha_n + (H_n\varepsilon^r_n + \varepsilon^o_n) \stackrel{\text{def}}{=} \mathcal{H}_n\alpha_n + \check{\varepsilon}^o_n, \quad n = 1, \ldots, N,$$

$$\alpha_{n+1} = \mathcal{A}_n\alpha_n + E^T\varepsilon^{\mathrm{m}}_n \stackrel{\text{def}}{=} \mathcal{A}_n\alpha_n + \check{\varepsilon}^{\mathrm{m}}_n, \quad n = 1, \ldots, N-1.$$

$$\mathcal{Q}_n = \langle \check{\varepsilon}^{\mathrm{m}}_n \check{\varepsilon}^{\mathrm{m}\,T}_n \rangle = E^T \langle \varepsilon^{\mathrm{m}}_n \varepsilon^{\mathrm{m}\,T}_n \rangle E = E^T Q_n E$$

$$\mathcal{R}_n = \langle \check{\varepsilon}^o_n \check{\varepsilon}^{o\,T}_n \rangle = \langle (H_n\varepsilon^r_n + \varepsilon^o_n)(H_n\varepsilon^r_n + \varepsilon^o_n)^T \rangle =$$
$$\langle \varepsilon^o_n \varepsilon^{o\,T}_n \rangle + H_n \langle \varepsilon^r_n \varepsilon^{r\,T}_n \rangle H^T_n \stackrel{\text{def}}{=} R_n + H_n Q^r H^T_n \stackrel{\text{def}}{=} R_n + R'_n.$$

REDUCED SPACE OPTIMAL ANALYSIS

Cost function:

$$\mathcal{S}[\alpha_1, \alpha_2, \ldots, \alpha_N] = \sum_{n=1}^{N} (\mathcal{H}\alpha_n - \mathcal{T}_n^o)^T \mathcal{R}_n^{-1} (\mathcal{H}\alpha_n - \mathcal{T}_n^o) + \sum_{n=1}^{N-1} (\alpha_{n+1} - \mathcal{A}_n\alpha_n)^T \mathcal{Q}_n^{-1} (\alpha_{n+1} - \mathcal{A}_n\alpha_n).$$

KF:

$$
\begin{aligned}
\alpha_n^a &= \alpha_n^f + \mathcal{K}_n \left(\mathcal{T}_n^o - \mathcal{H}_n \alpha_n^f \right), \\
\alpha_n^f &= \mathcal{A}_n \alpha_{n-1}^a, \\
\mathcal{K}_n &= \left(\mathcal{H}_n^T \mathcal{R}_n^{-1} \mathcal{H}_n + \mathcal{P}_n^{f\ -1} \right)^{-1} \mathcal{H}_n^T \mathcal{R}_n^{-1} \\
\mathcal{P}_n^a &= (I_L - \mathcal{K}_n \mathcal{H}_n) \mathcal{P}_n^f \\
\mathcal{P}_n^f &= \mathcal{A}_{n-1} \mathcal{P}_{n-1}^a \mathcal{A}_{n-1}^T + \mathcal{Q}_{n-1}, \qquad n = 2, 3, \dots, N
\end{aligned}
$$

OS:

$$
\begin{aligned}
\alpha_n^s &= \alpha_n^a + G_n \left(\alpha_{n+1}^s - \mathcal{A}_n \alpha_n^a \right), \\
G_n &= \mathcal{P}_n^a \mathcal{A}_n^T (\mathcal{P}_{n+1}^f)^{-1}, \\
\mathcal{P}_n^s &= \mathcal{P}_n^a + G_n \left(\mathcal{P}_{n+1}^s - \mathcal{P}_{n+1}^f \right) G_n^T, \qquad n = N-1, \dots, 2, 1
\end{aligned}
$$

OI:

$$\mathcal{S}_n^{\mathrm{OI}}[\alpha_n] = (\mathcal{H}_n\alpha_n - \mathcal{T}_n^o)^T\mathcal{R}_n^{-1}(\mathcal{H}_n\alpha_n - \mathcal{T}_n^o) + \alpha_n^T\mathcal{C}_n^{-1}\alpha_n, \qquad n > 1$$

$$\alpha_n^{\mathrm{OI}} = \mathcal{P}_n^{\mathrm{OI}}\mathcal{H}_n^T\mathcal{R}_n^{-1}\mathcal{T}_n^o, \quad \mathcal{P}_n^{\mathrm{OI}} = \left(\mathcal{H}_n^T\mathcal{R}_n^{-1}\mathcal{H}_n + \mathcal{C}_n^{-1}\right)^{-1}.$$

Projection:

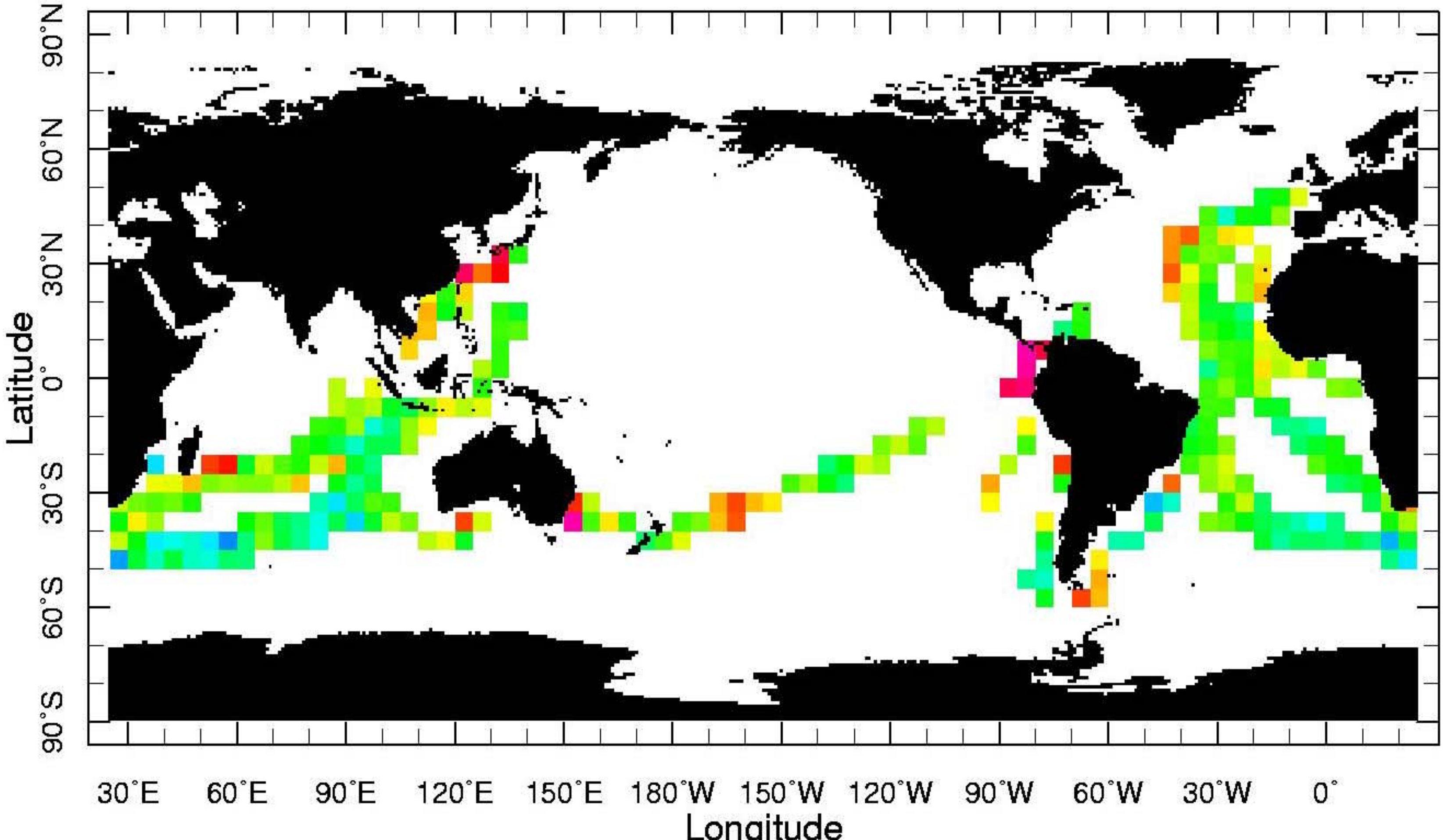
$$\alpha_n^p = \mathcal{P}_n^p\mathcal{H}_n^T\mathcal{R}_n^{-1}\mathcal{T}_n^o, \qquad \mathcal{P}_n^p = (\mathcal{H}_n^T\mathcal{R}_n^{-1}\mathcal{H}_n)^{-1}$$

Spagetti-western properties of least-squares estimates of spectrally red signals: (good) can be approximated by a few modes, (bad) have less variance than the true signal, and (ugly) redder than the true signal.

It helps to remember that

$$C=<TT^T>+P$$

Dec 1868: Available observations



Dec 1868

ABSTRACT LOG

RECOMMENDED BY THE

MARITIME CONFERENCE OF BRUSSELS,

Kept on board the U. S.

Commander, during the years 185

Indexed.

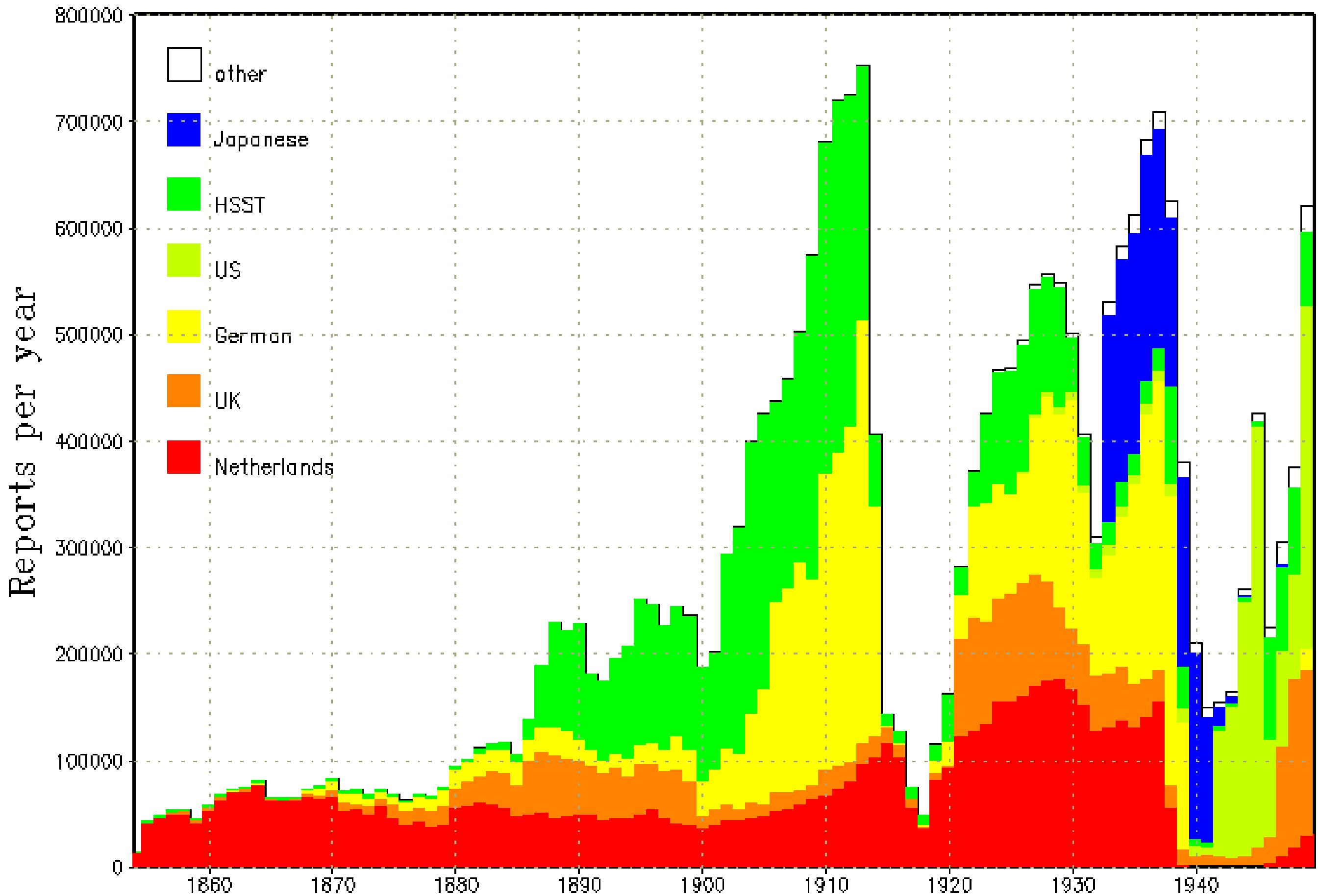
GENERAL ORDER.

Navy Department,

Signed, J. C. DOBBIN,

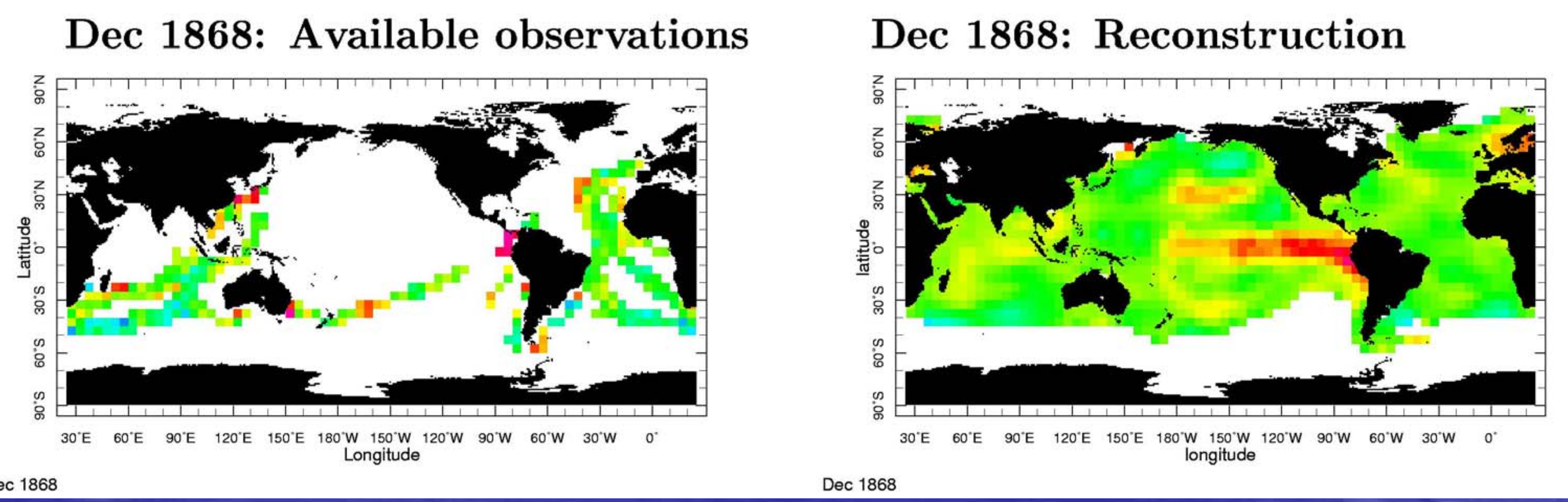
Secretary of the Navy.

Number of observations in ICOADS



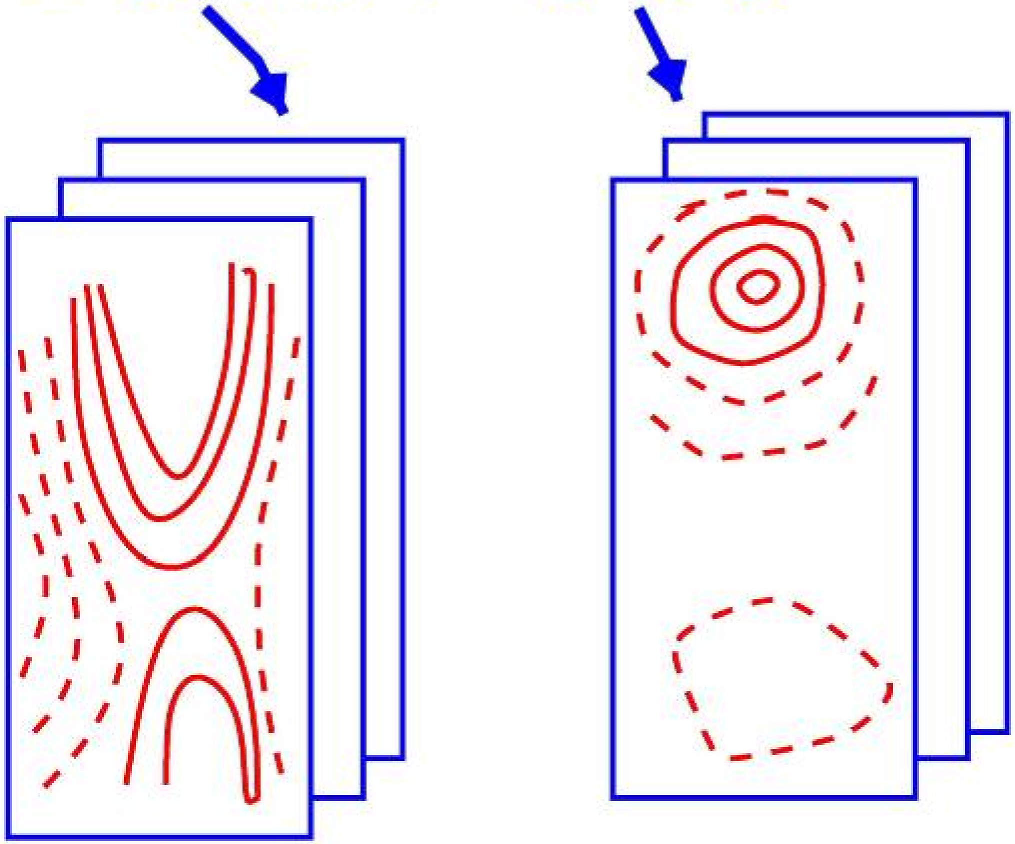
http://icoads.noaa.gov/

Given a choice, climatologists in general would rather use the righthand panel below than the lefthand one



APPROXIMATING COVARIANCE

$$C = E\Lambda E^T + E'\Lambda' E'^T$$

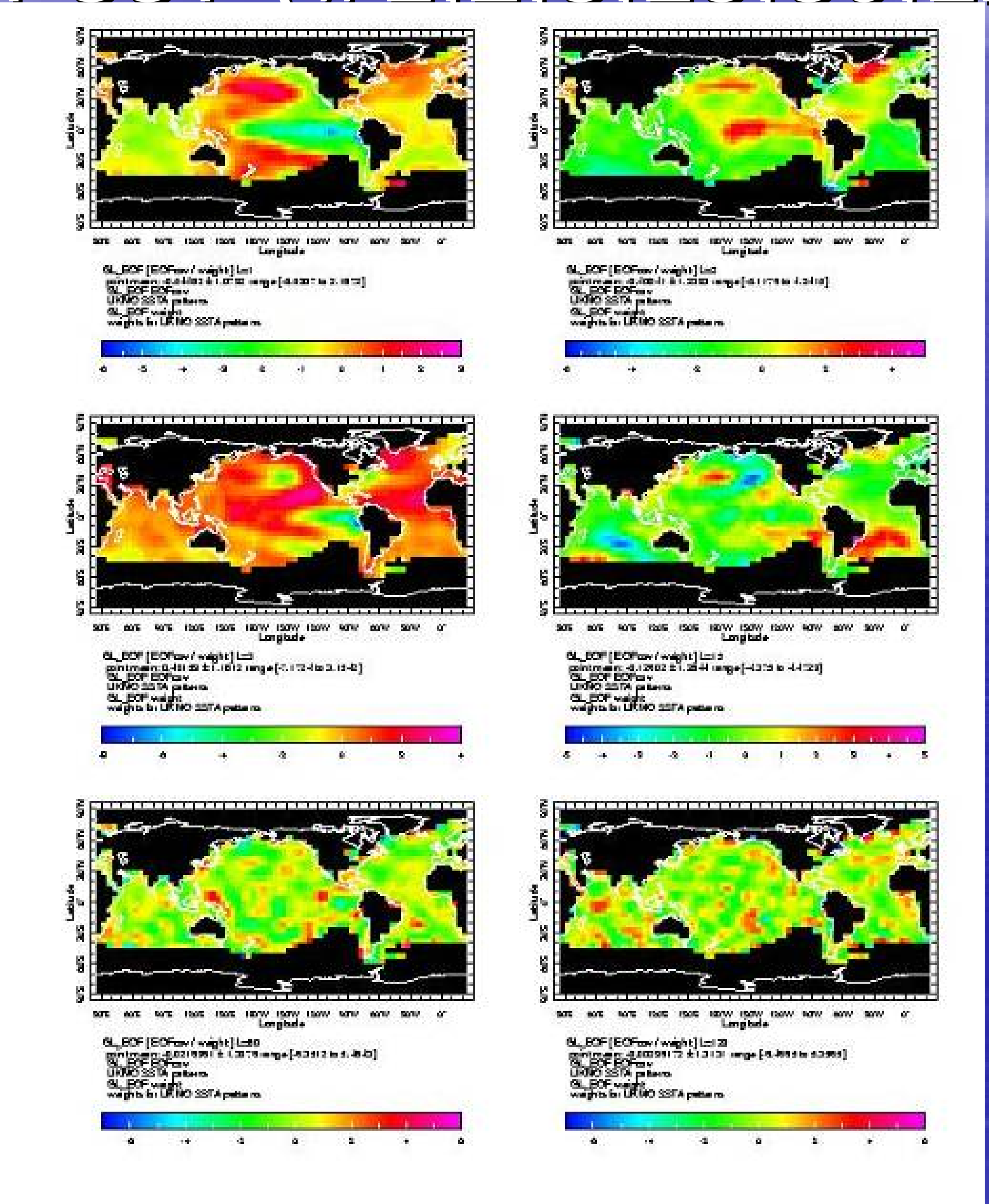


Reduced space optimal analysis

Successive corrections; Kriging

EOFs of SST (#1,2,3,15,80,120)

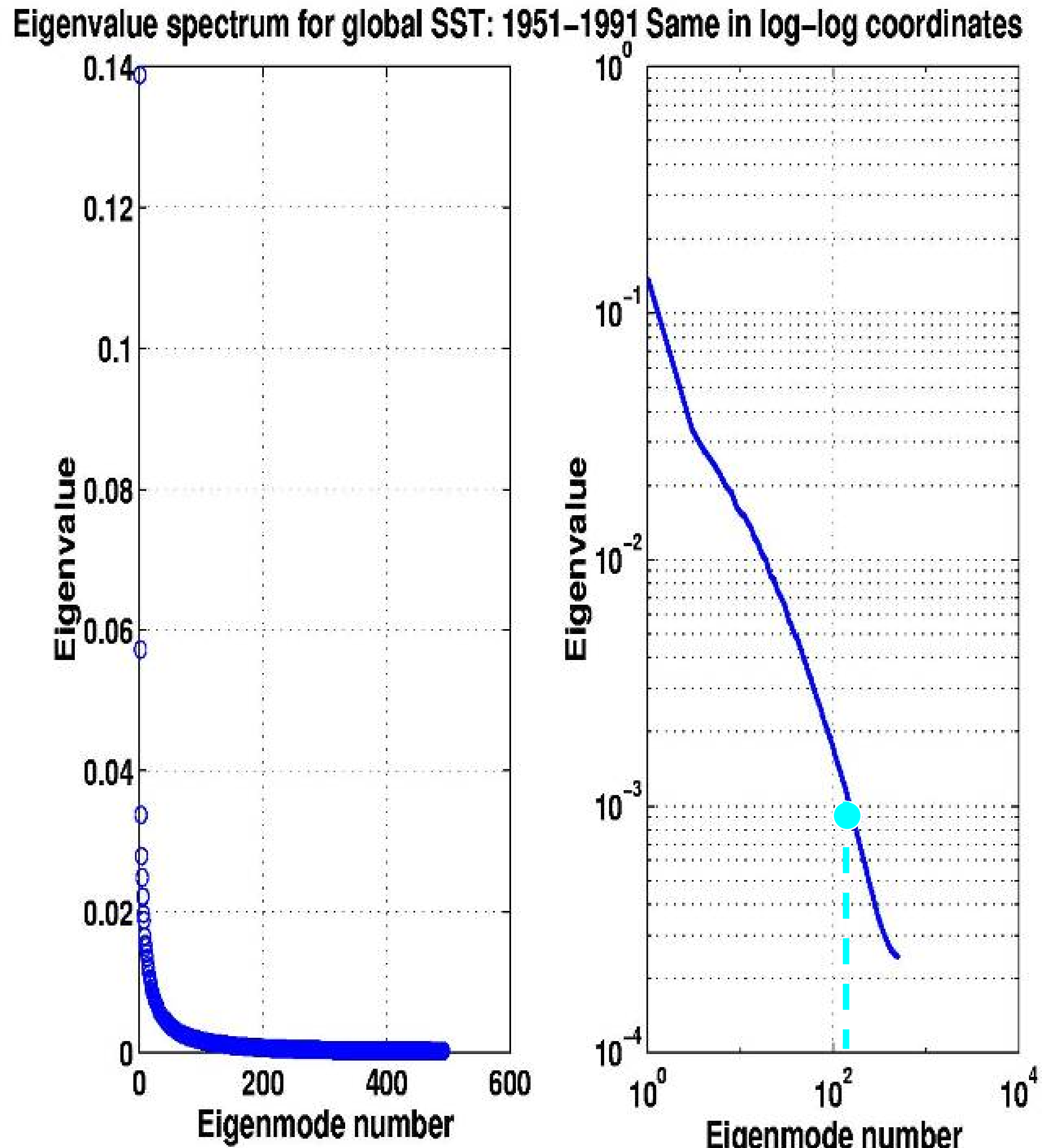
EOF 1
14%



EOF 15
1%

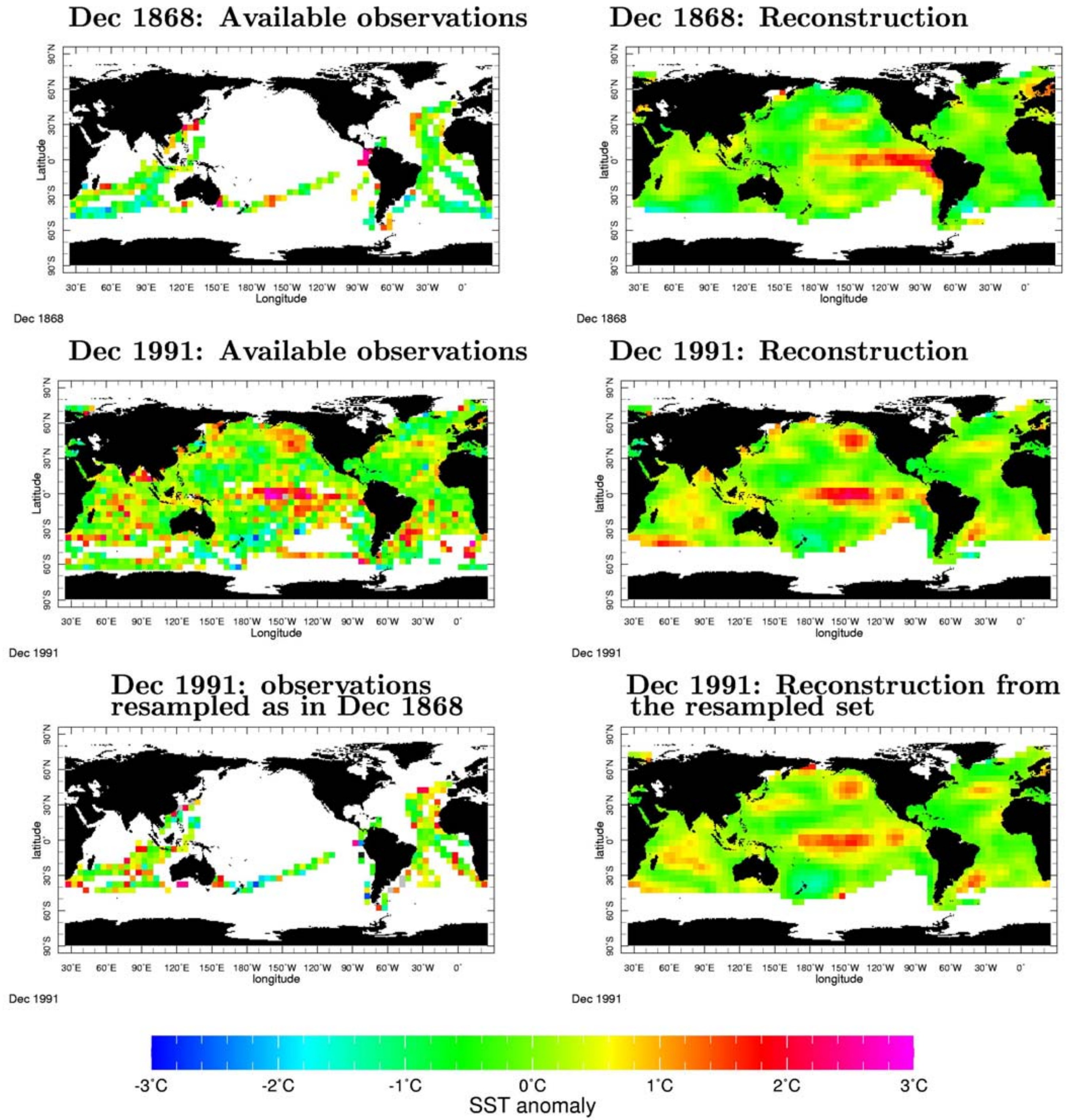
EOF 80
0.1%

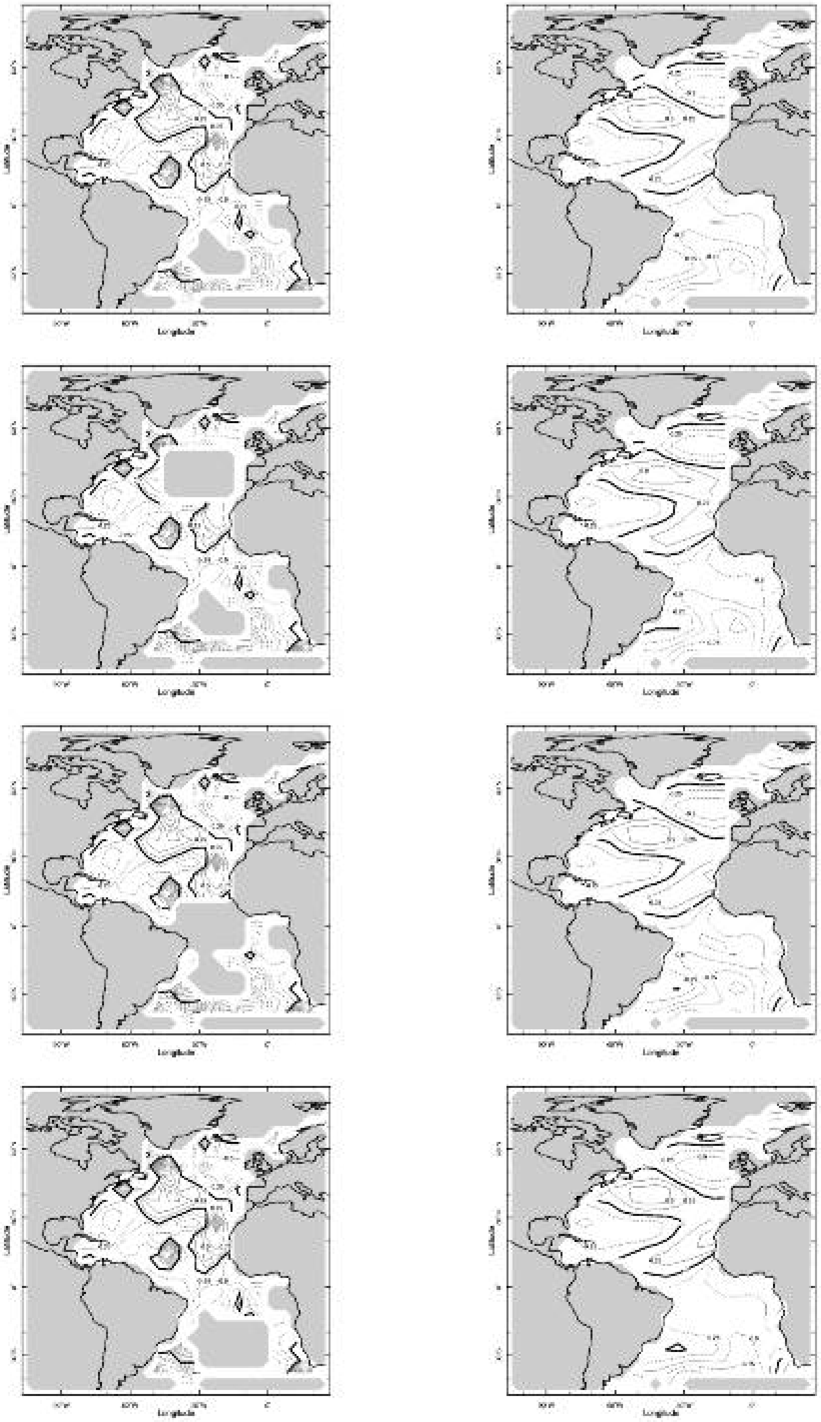
EOF 120
0.02%



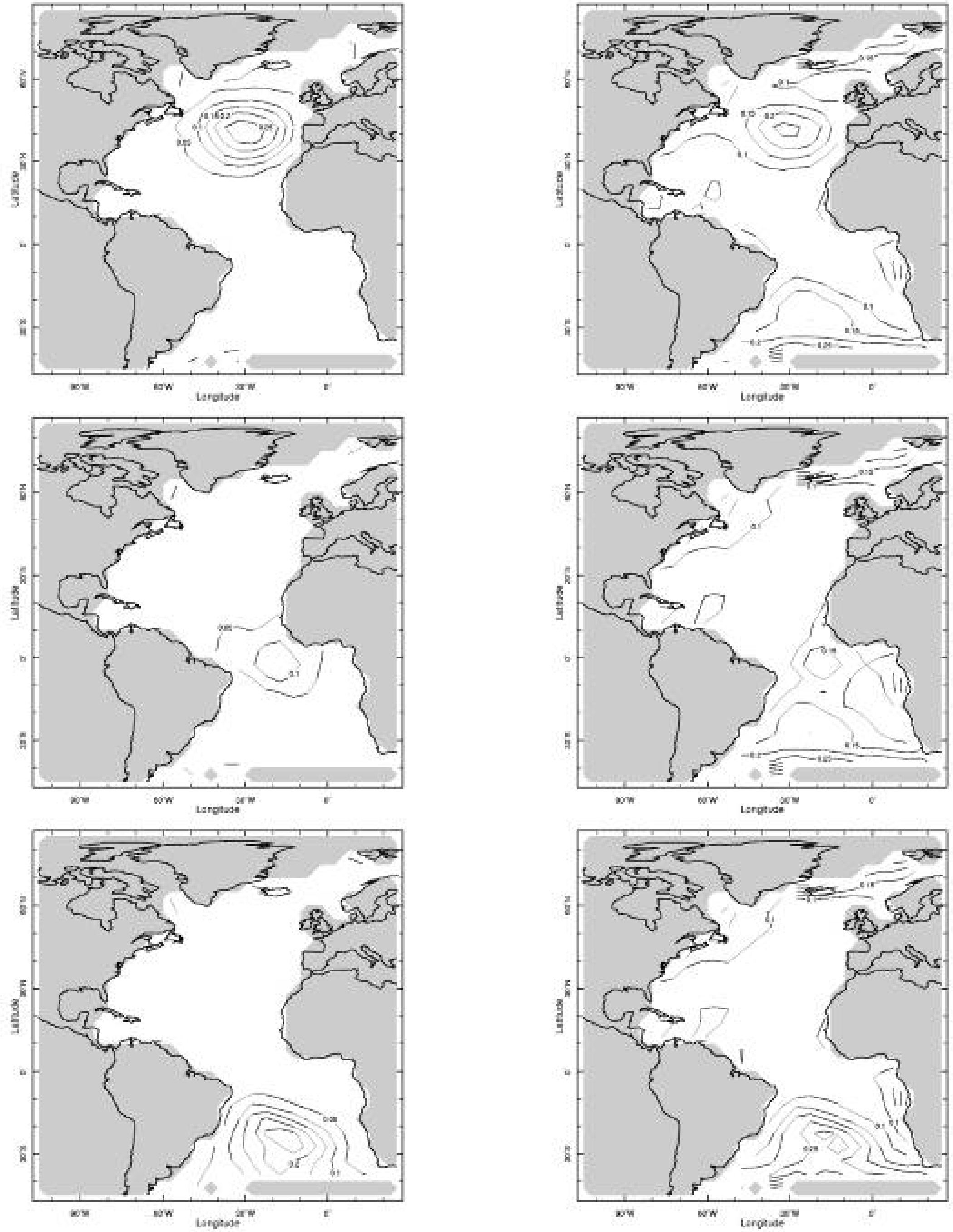
Eigenvalue spectrum for global SST: 1951-1991 Same in log-log coordinates
Eigenvalue
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
200
400
600
Eigenmode number
Eigenvalue
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^0
10^2
10^4
Eigenmode number

VERIFICATION OF REDUCED SPACE SST ANALYSES


Dec 1868: Available observations
Dec 1868: Reconstruction
Dec 1991: Available observations
Dec 1991: Reconstruction
Dec 1991: observations resampled as in Dec 1868
Dec 1991: Reconstruction from the resampled set
Latitude
latitude
Longitude
longitude
90°N 60°N 30°N 0° 30°S 60°S 90°S
30°E 60°E 90°E 120°E 150°E 180°W 150°W 120°W 90°W 60°W 30°W 0°
Dec 1868
Dec 1991
-3°C -2°C -1°C 0°C 1°C 2°C 3°C
SST anomaly



Actual rms difference Theoretical error



El Niño of 1877-1878 in analyzed anomalies

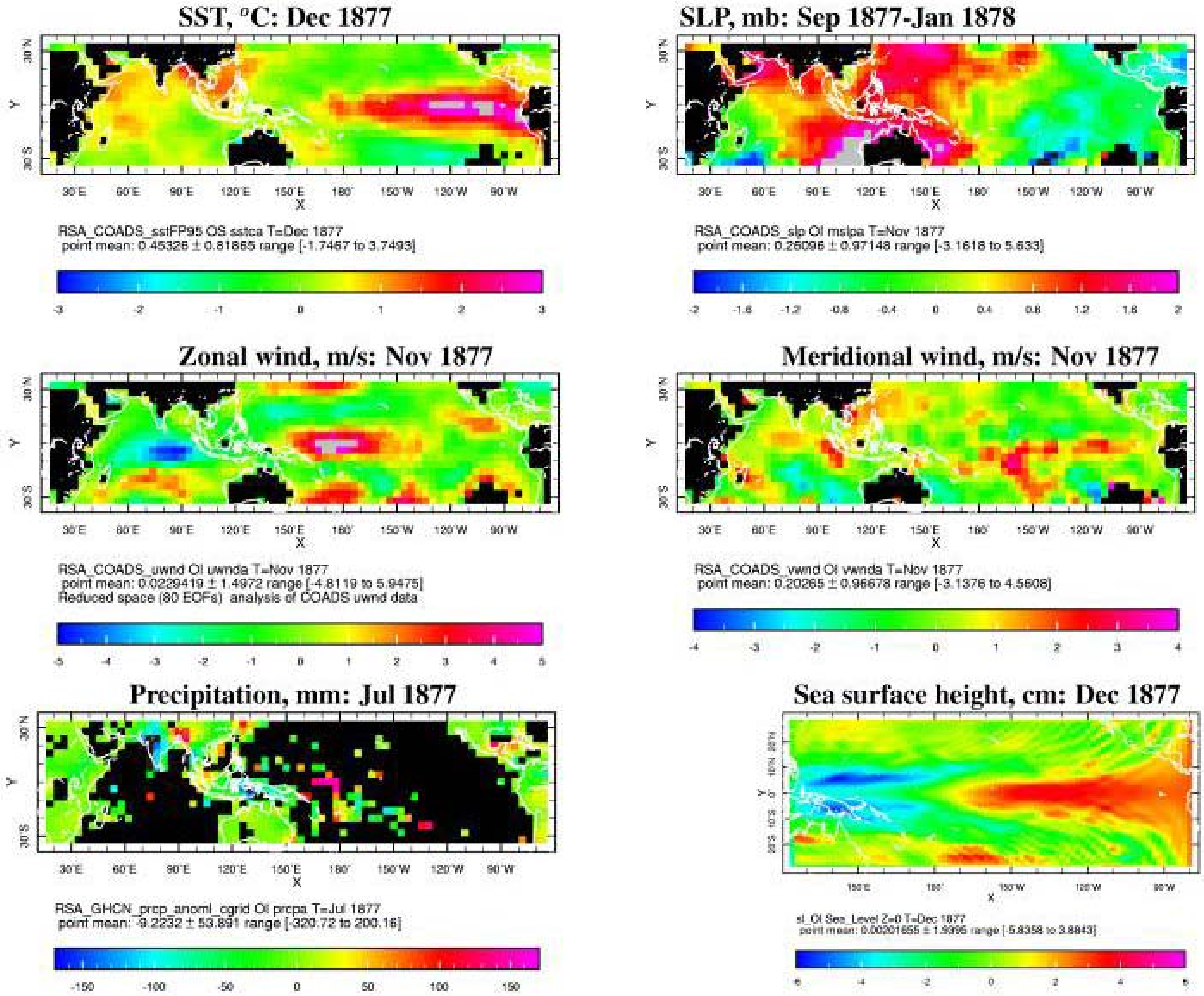
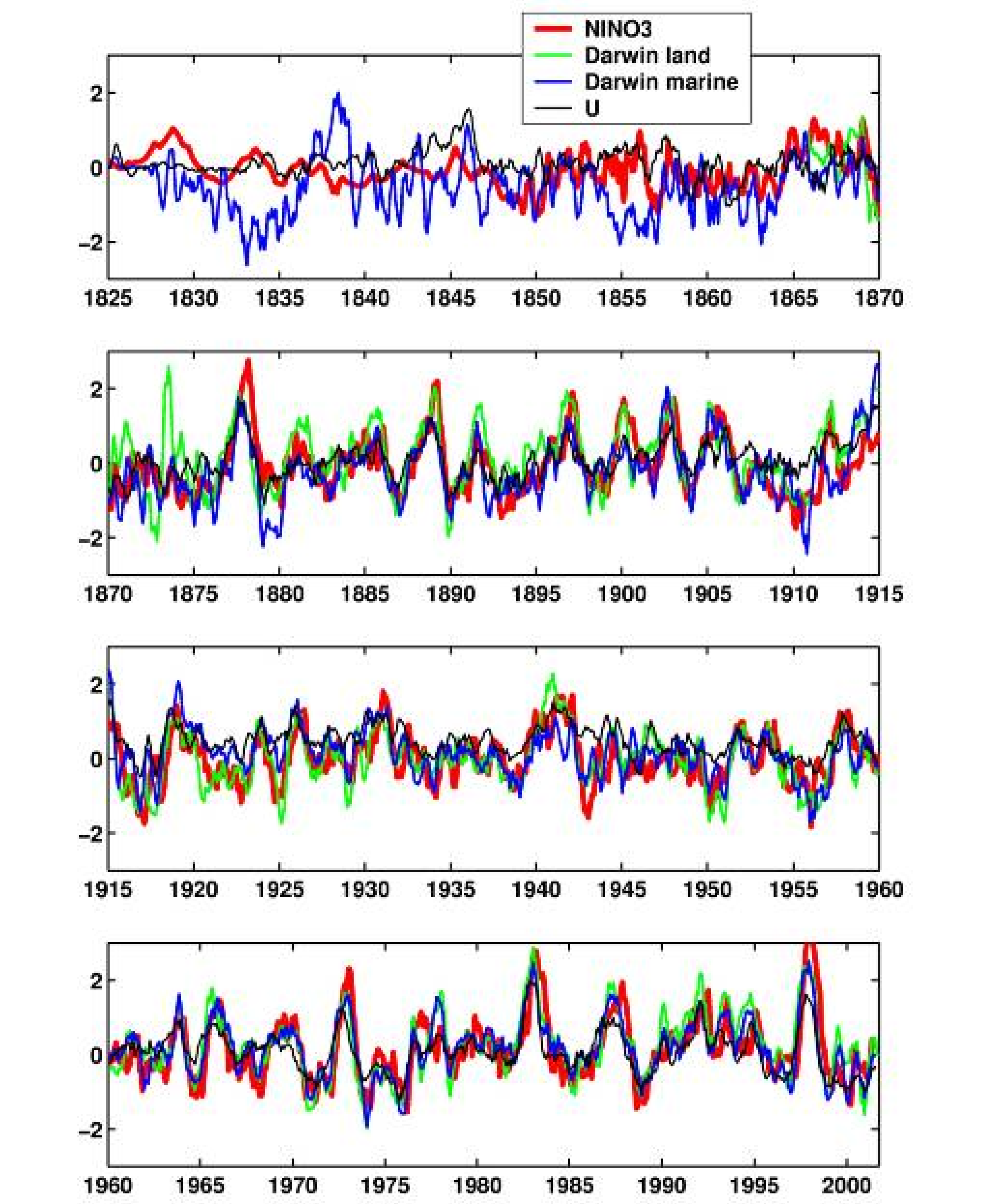


Figure 4: Anomalies of 1877-1878 El Niño illustrated by univariate reduced analyses by *Kaplan et al.* [2001b]

Independent ENSO indices



LDEO5

Chen et al., 2004

NINO3 (SST 5S-5N,150W-90W)

1877-1878

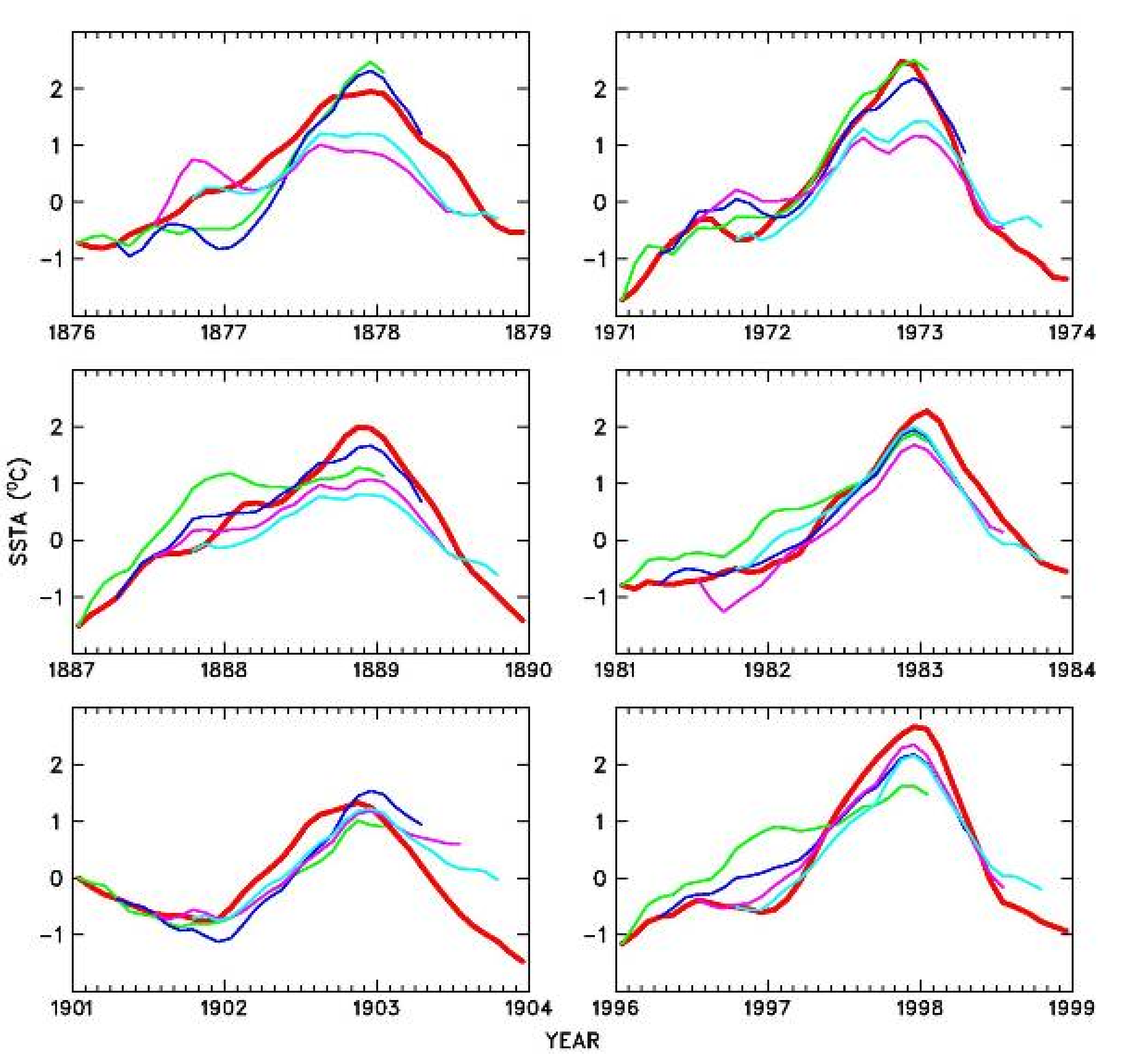
1972-1973

1888-1889

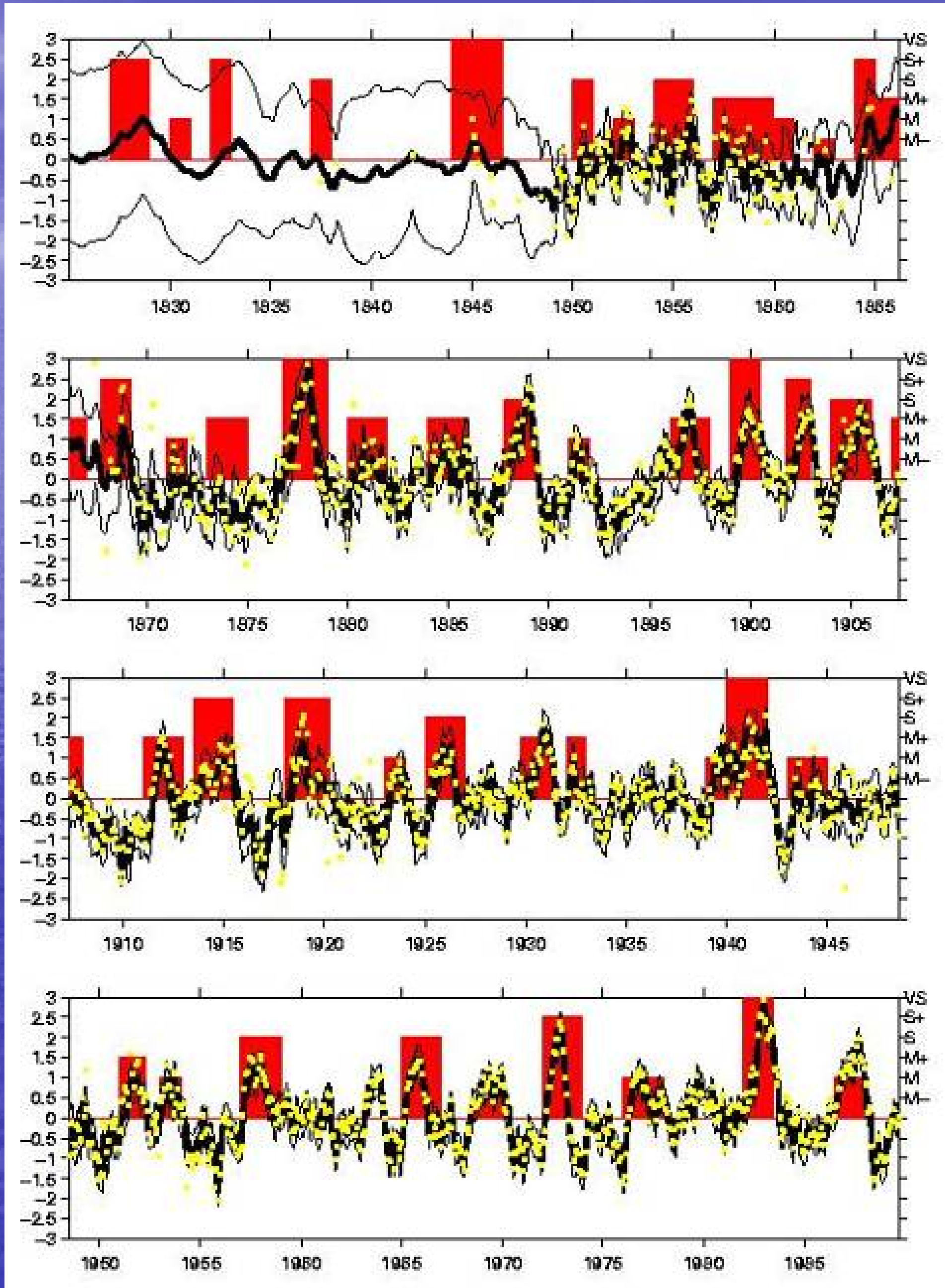
1982-1983

1902-1903

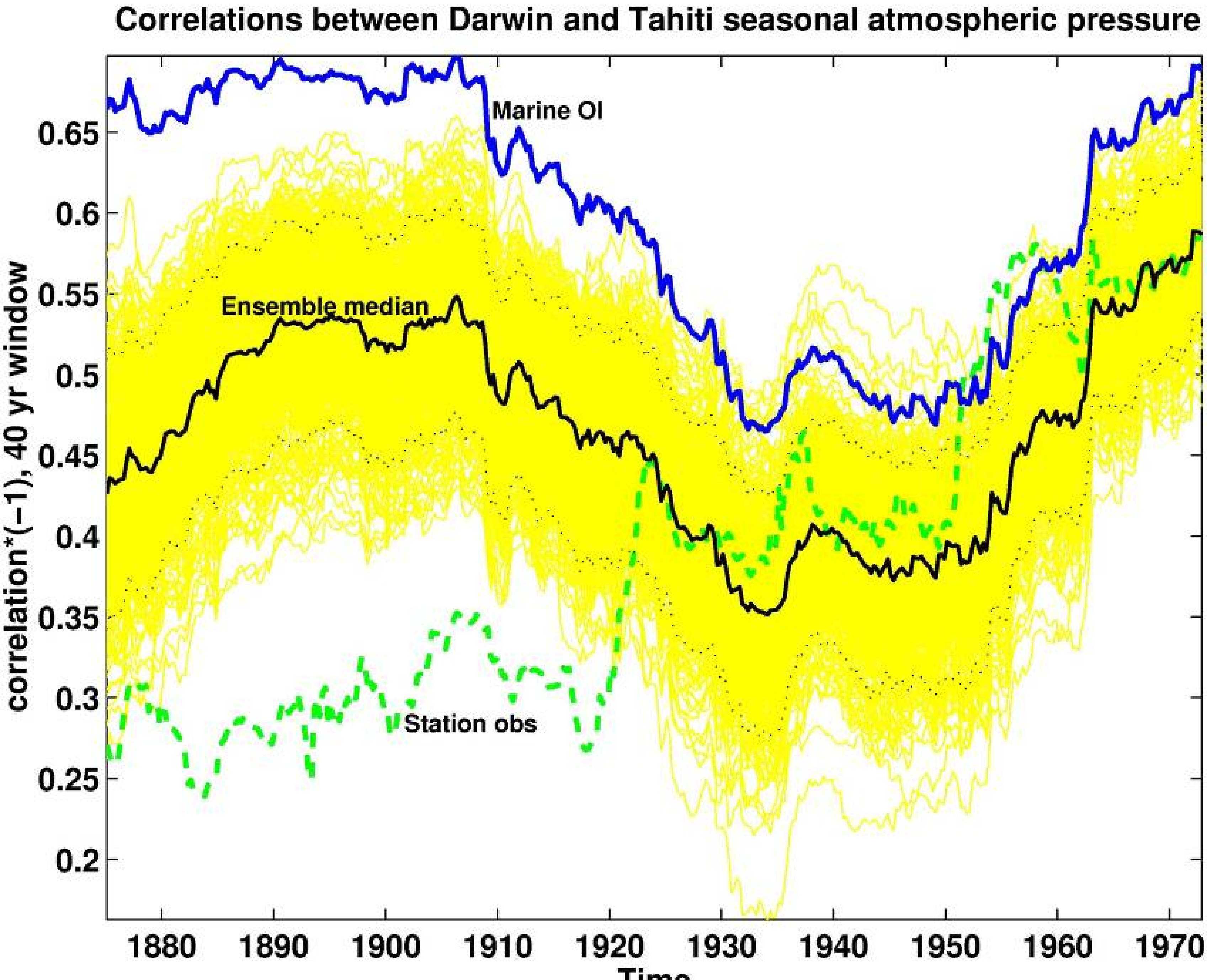
1997-1978



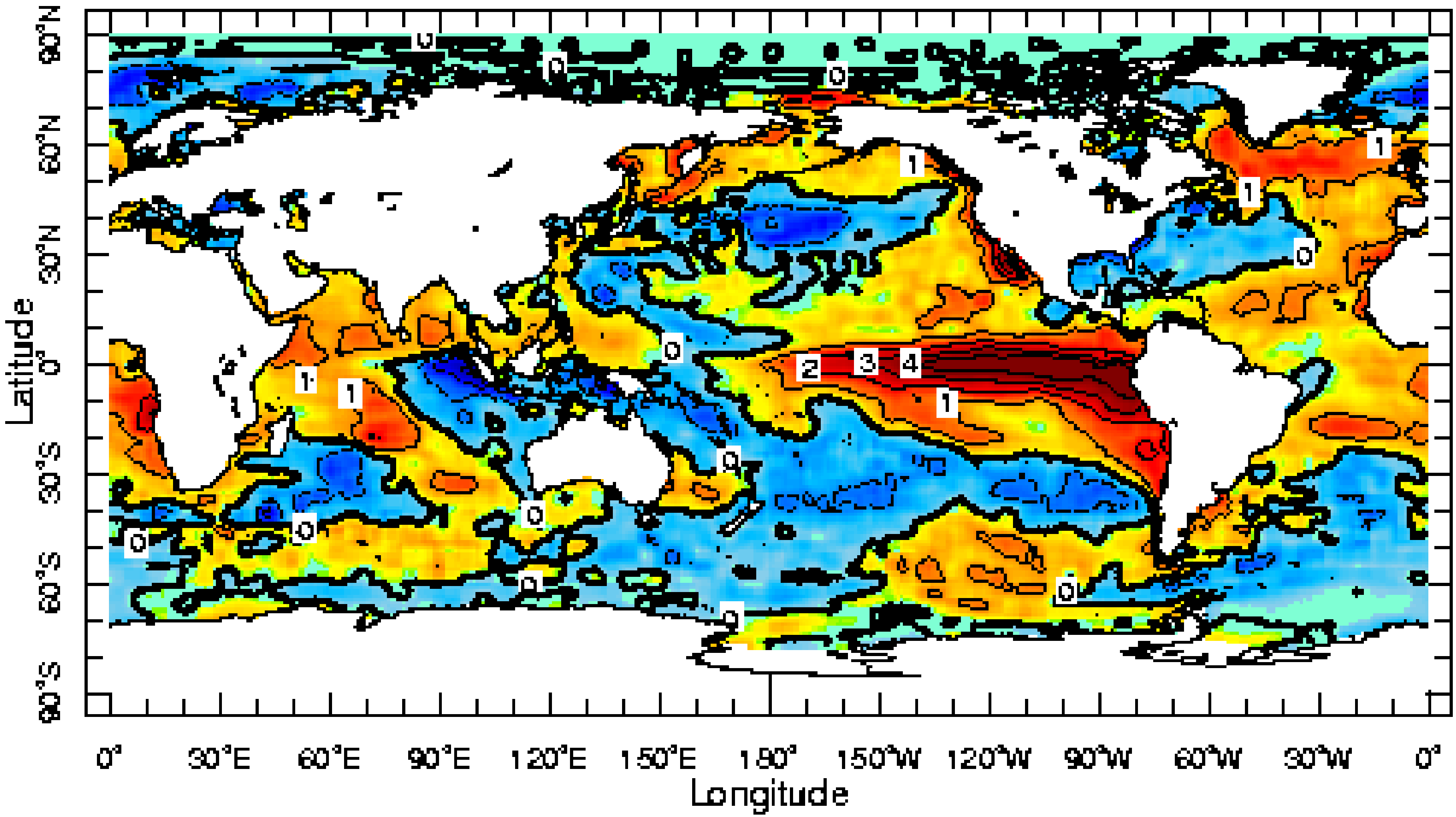
SST El Nino indices vs Quinn's historical rankings



OUTSTANDING PROBLEMS

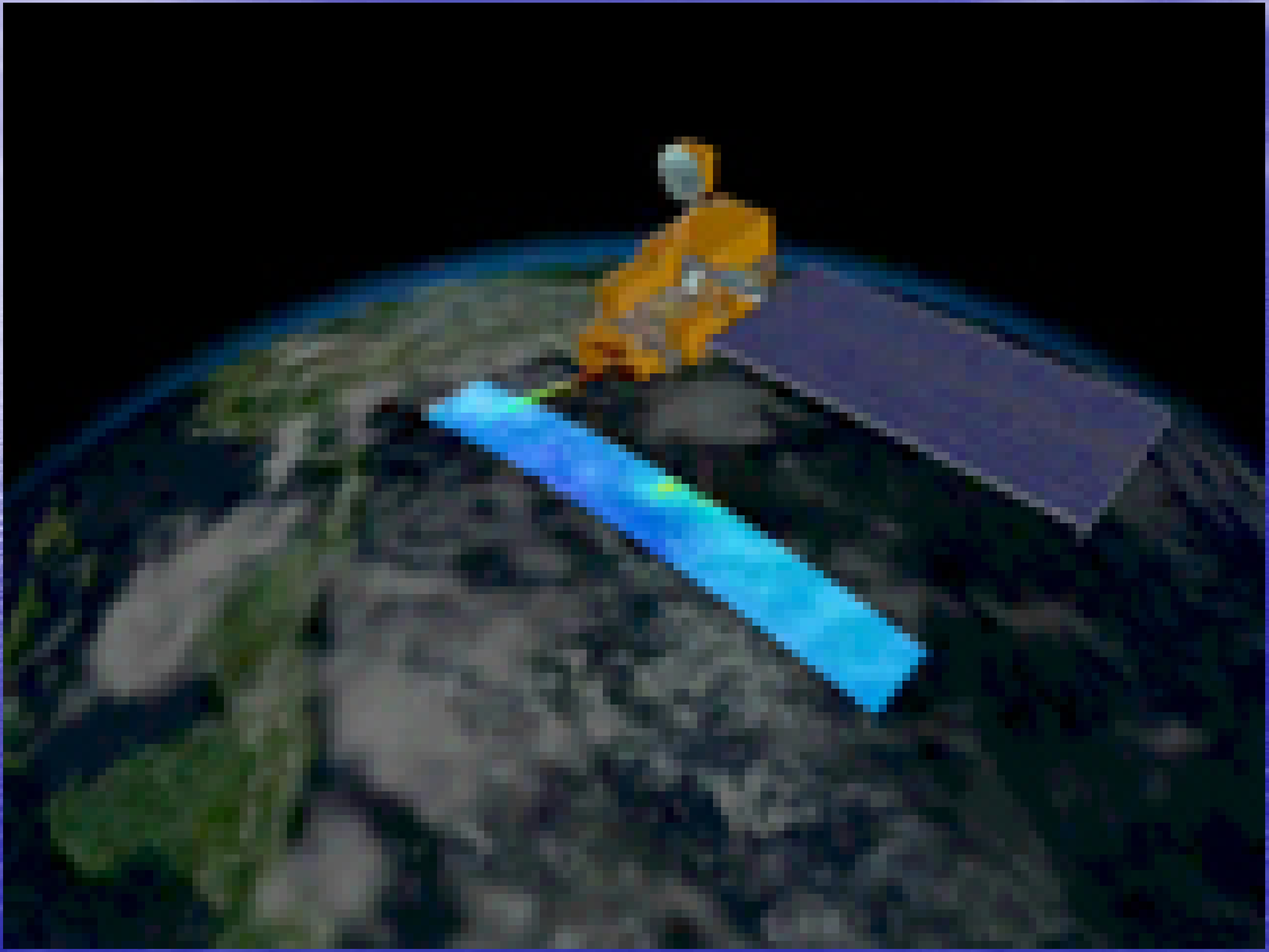

Correlations between Darwin and Tahiti seasonal atmospheric pressure
Marine OI
Ensemble median
Station obs
correlation*(−1), 40 yr window
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
Time

Sea Surface Temperature Anomaly (Reynolds and Smith's NCEP OI v.2)

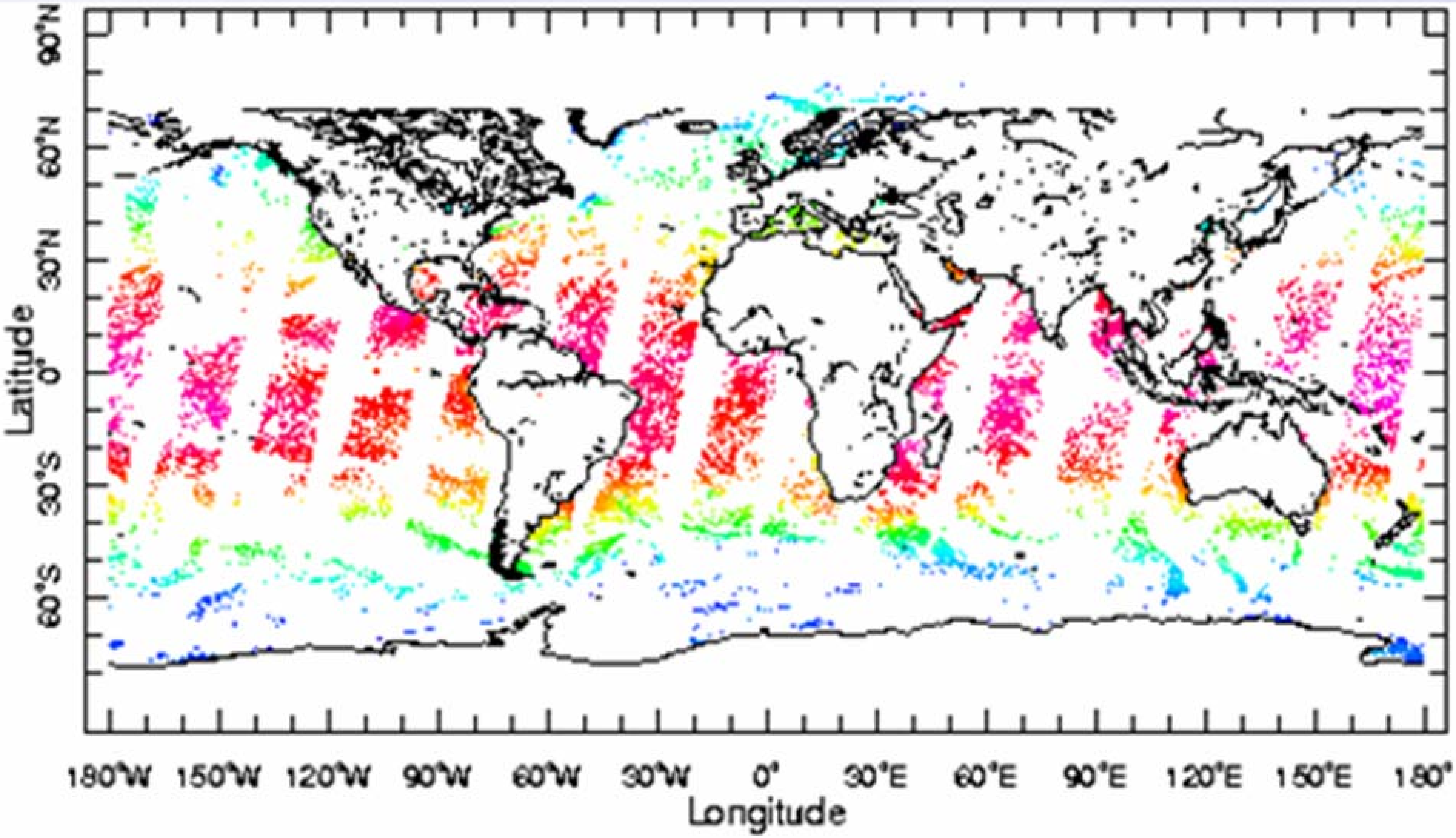


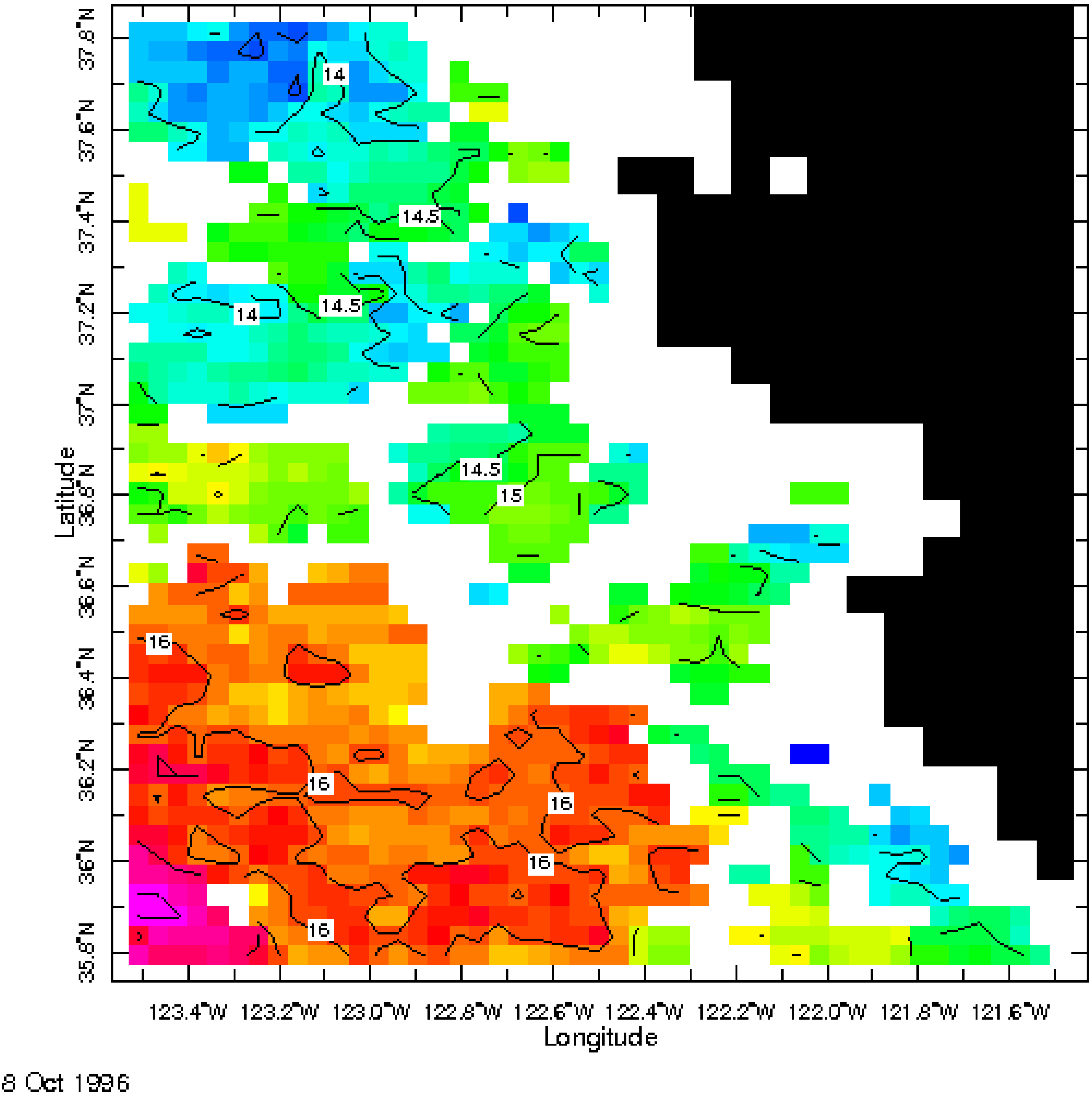
9-15 Nov 1997

MODIS Scanning Swath



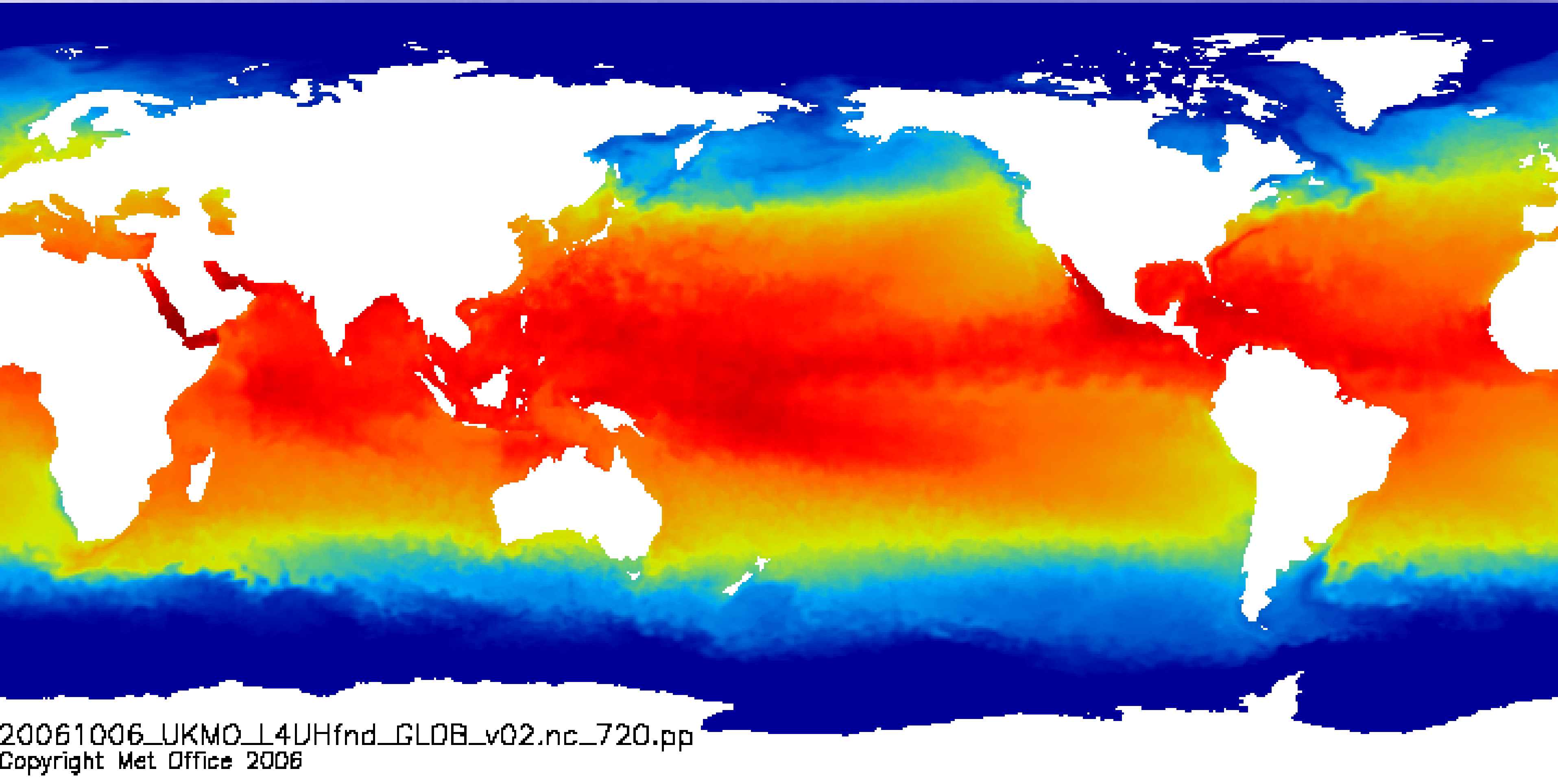
Satellite Sea Surface Temperature Measurements for one day





Pathfinder SST: Monterey Bay, Oct 8, 1996 4km resolution

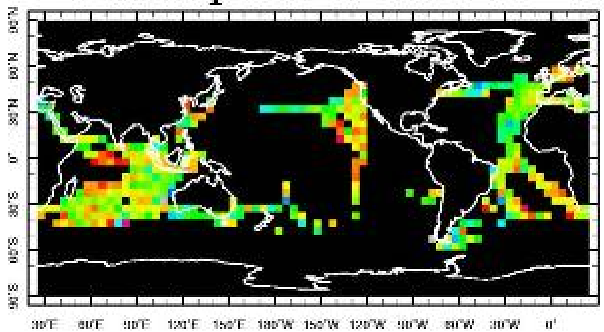
Operational Sea Surface Temperature and Sea Ice Analysis (OSTIA), from U.K. Met Ofice and GHRSST, blend of many satellite data streams



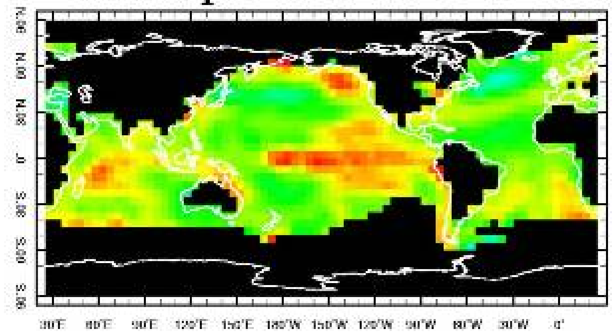
INVESTIGATION OF WHAT IS “LOST”:

ERROR OF TRUNCATION

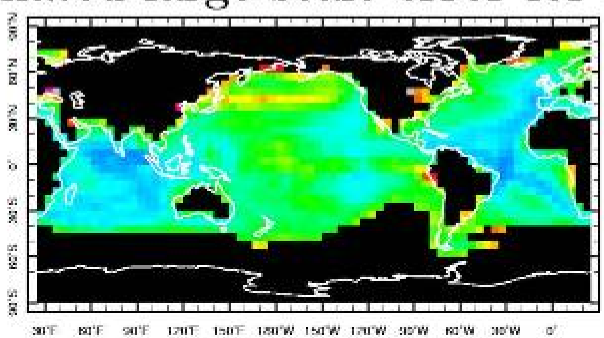
(e) Observations of Dec 1986 resampled as in Dec 1877



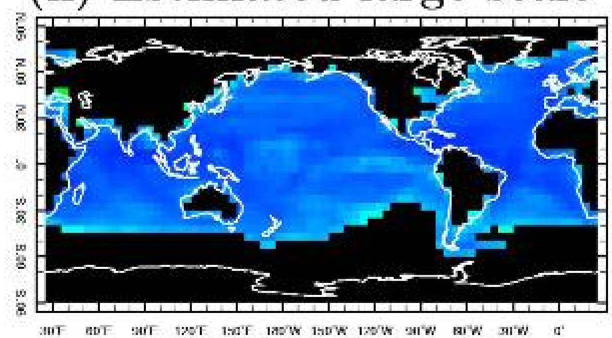
(f) Analysis of Dec 1986 for obs resampled as in Dec 1877



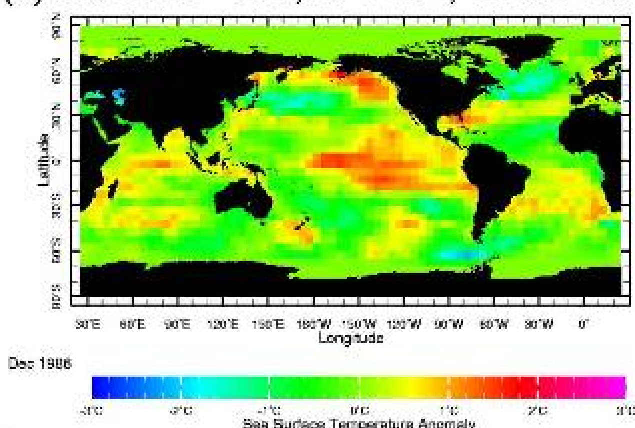
(g) Estimated large scale error for 1877



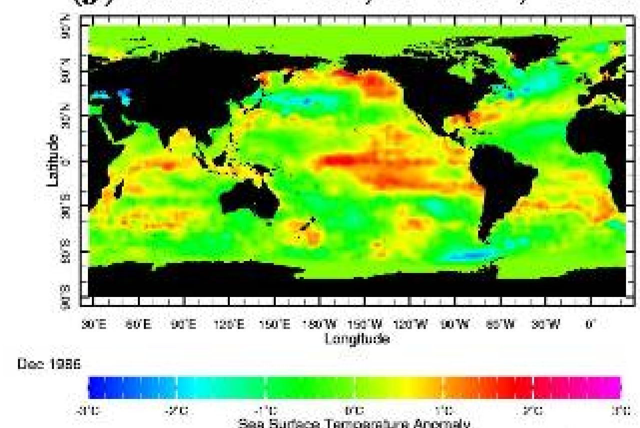
(h) Estimated large scale error for 1986



(i) NCEP OI, $5^o \times 5^o$; Dec 1986

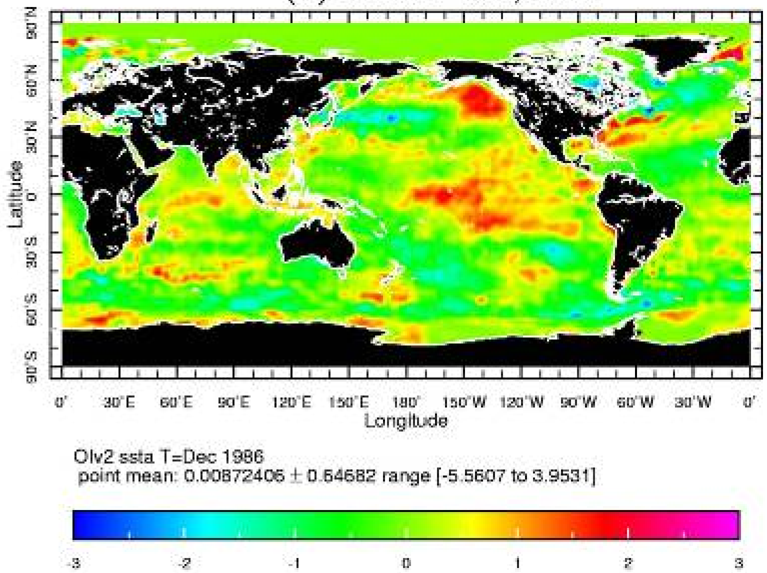


(j) NCEP OI, $1^o \times 1^o$; Dec 1986

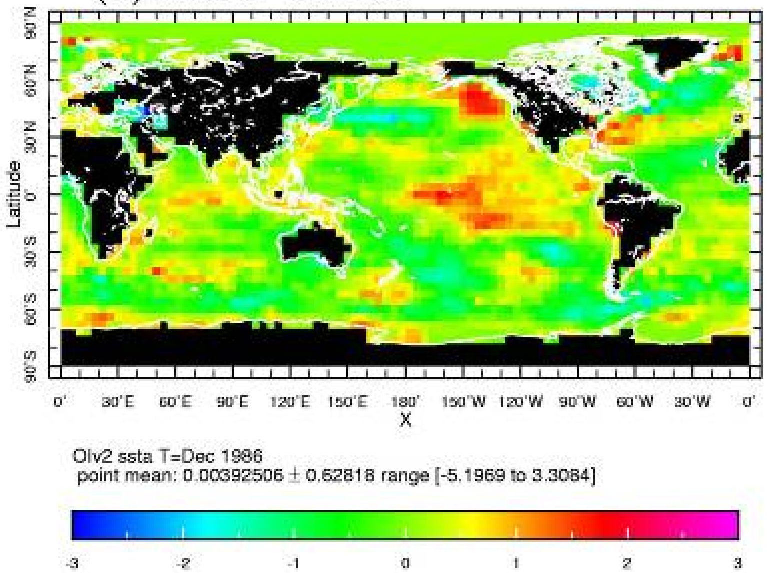


December 1986

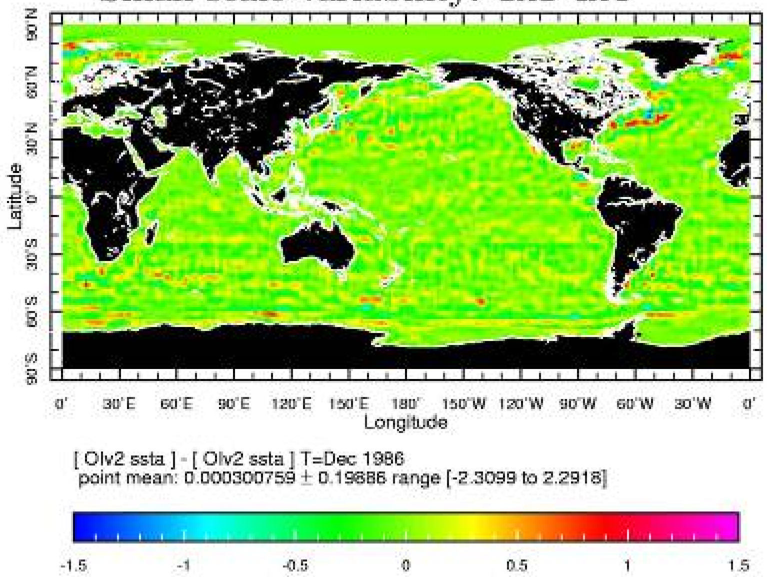
(a) NCEP OI, 1x1



(b) NCEP OI: 4x4

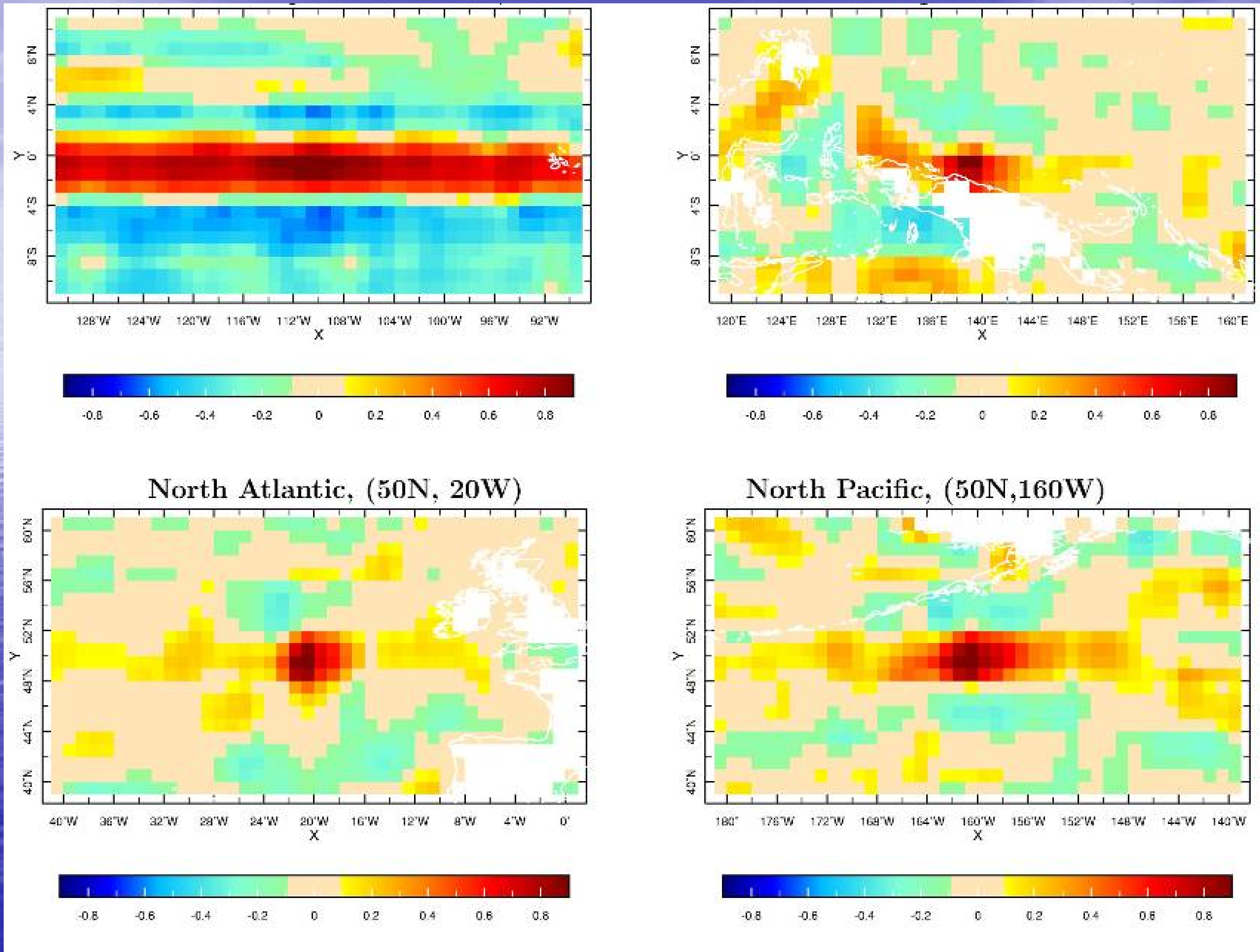


Small-scale variability: 1x1-4x4



Truncation error autocovariance patterns:

Eastern Equatorial Pacific at 110W   Western Equatorial Pacific at 140E



Scale separation in a field estimate

OI problem: estimating a field $\mathcal{T}$ from a first-guess (background) solution $\mathcal{T}^b$ and an incomplete set of observations $\mathcal{T}^o$ is given by:

$$\mathcal{T}^b = \mathcal{T} + \varepsilon^b, \;\; \langle \varepsilon^b \rangle = 0, \;\; \langle \varepsilon^b \varepsilon^{b\,T} \rangle = C,$$
$$\mathcal{T}^o = H\mathcal{T} + \varepsilon^o, \;\; \langle \varepsilon^o \rangle = 0, \;\; \langle \varepsilon^o \varepsilon^{o\,T} \rangle = R.$$

The solution to this OI problem is a minimizer $\hat{\mathcal{T}}$ of the cost function

$$\mathbf{S}[\mathcal{T}] = (H\mathcal{T} - \mathcal{T}^o)^T R^{-1} (H\mathcal{T} - \mathcal{T}^o) + (\mathcal{T} - \mathcal{T}^b)^T C^{-1} (\mathcal{T} - \mathcal{T}^b).$$

$$\hat{\mathcal{T}} = CH^T (R + HCH^T)^{-1} \mathcal{T}^o.$$

$$C = E \Lambda E + E' \Lambda' E' = E \Lambda E + C'$$

$$\hat{\mathcal{T}} = E\hat{\alpha} + \Delta\hat{\mathcal{T}}.$$

$$\mathcal{T}^o = HE\alpha + \breve{\varepsilon}^o, \qquad \langle \alpha\alpha^T \rangle = C', \qquad \langle \breve{\varepsilon}^o \breve{\varepsilon}^{o\,T} \rangle = HC'H^T + R.$$

$$\hat{\alpha} = \Lambda E^T H^T (HE\Lambda E^T H^T + HC'H^T + R)^{-1} \mathcal{T}^o.$$

Observational residual: $\Delta\mathcal{T}^o = \mathcal{T}^o - HE\hat{\alpha}$

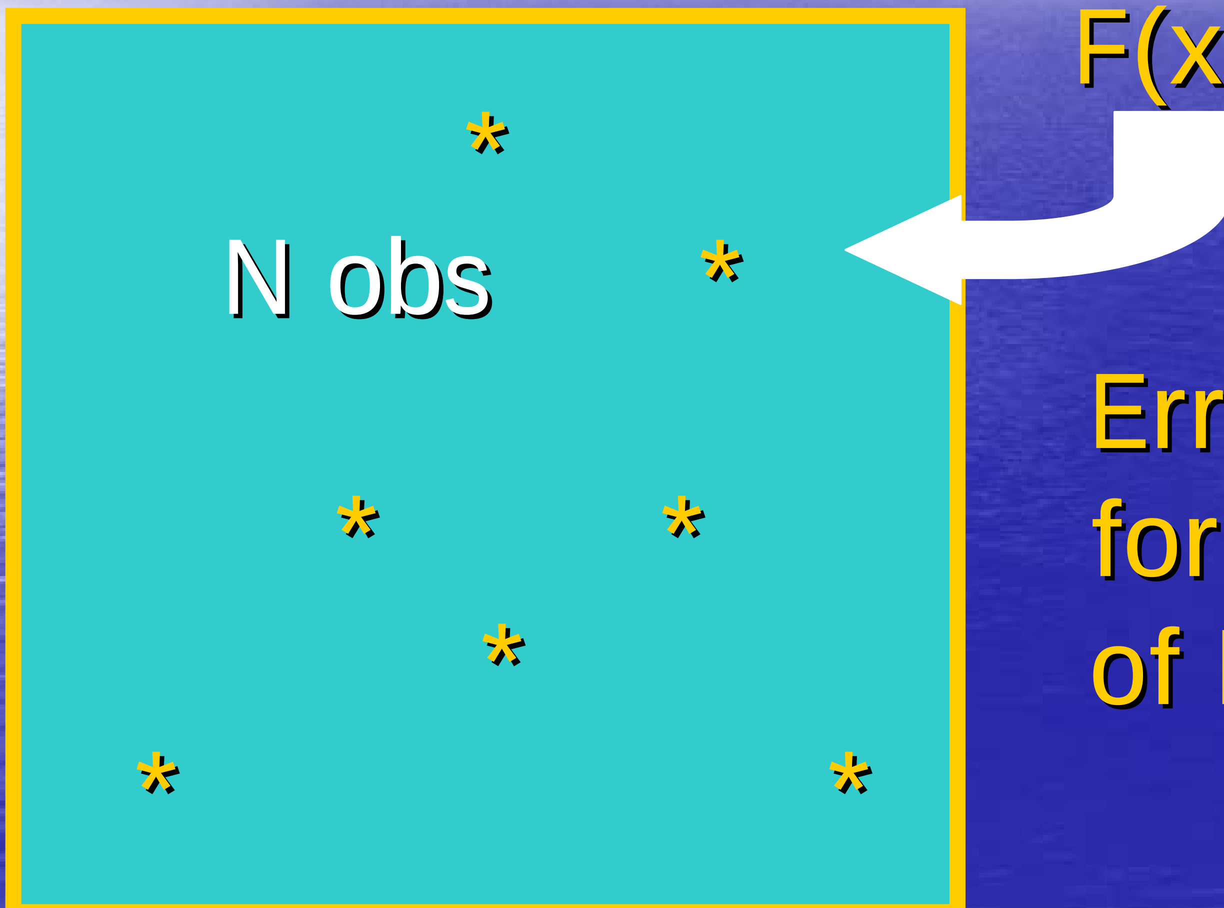
$$\Delta\mathcal{T}^o = H\Delta\mathcal{T} + \varepsilon^o, \qquad \langle \Delta\mathcal{T}\Delta\mathcal{T}^T \rangle = C', \qquad \langle \varepsilon^o \varepsilon^{o\,T} \rangle = R.$$

$$\Delta\mathcal{T} = C'H(HC'H^T + R)^{-1}\Delta\mathcal{T}^o$$

OBSERVATIONAL ERROR OF IN SITU DATA

What is the error in the binned obs mean (as estimates of the “true” bin area average)?

F(x,y) [or F(x,y,t)]



Error variance for the mean of N observ is

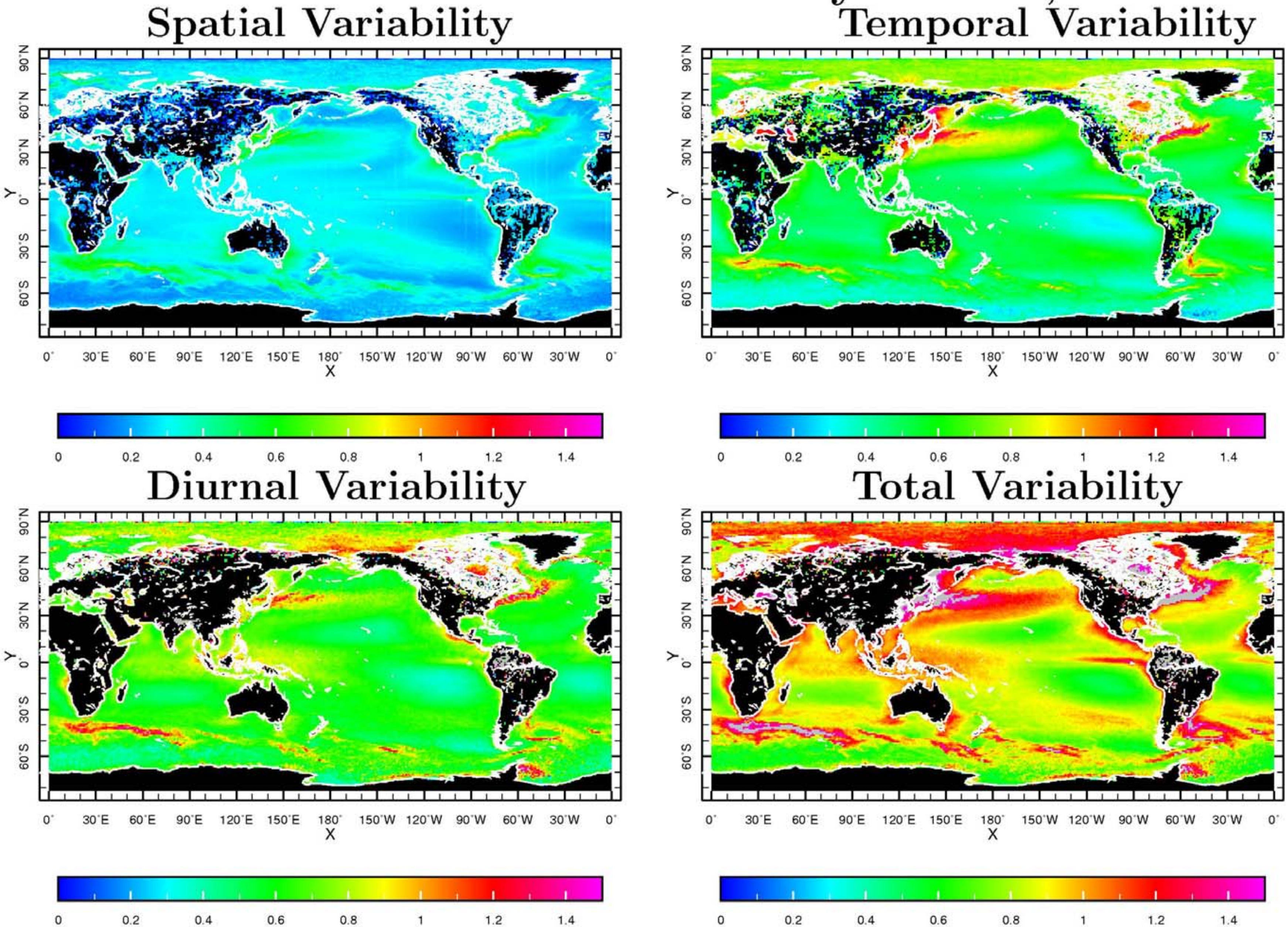
$$\sigma^2/N$$

High spatial and temporal resolution of satellite data can help pinpoint natural SST variability on small scales (below 1 deg) and short terms (within 1 month).

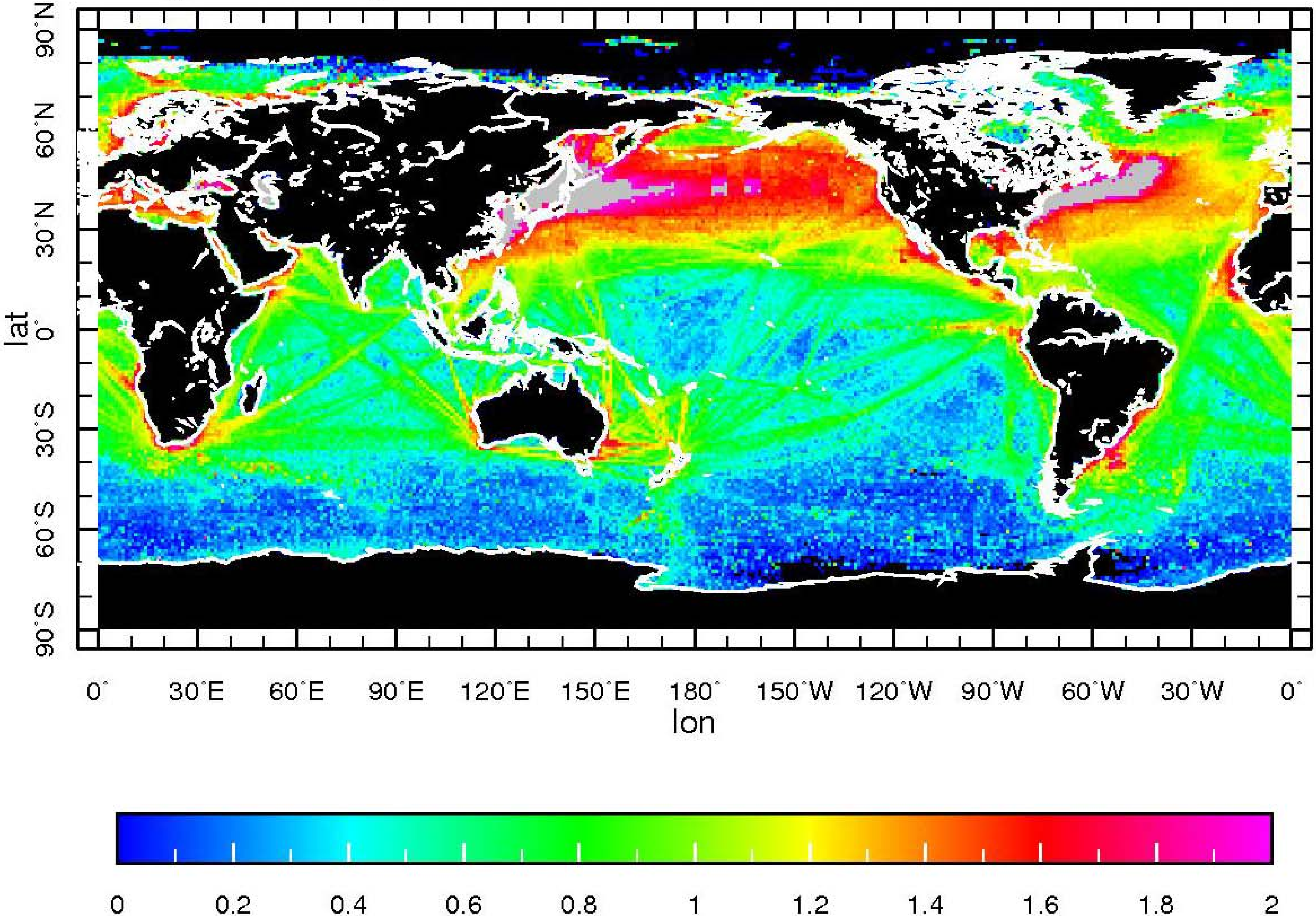
A few weeks of background processing of 20 years of daily 4km maps of Pathfinder SST gave us the SST variability inside 1x1 monthly boxes estimated.

[http://rainbow.ldeo.columbia.edu/~alexeyk/Satellite_SST.html]

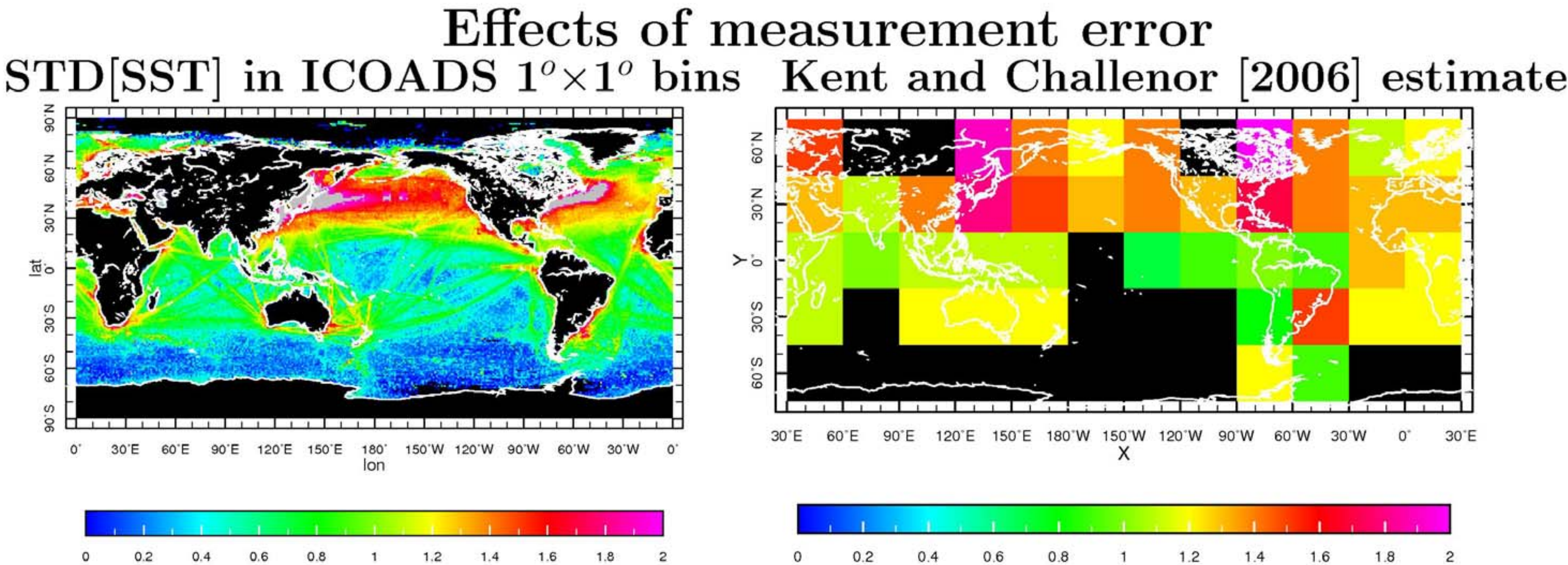
Small-scale variability in SST, ^{o}C



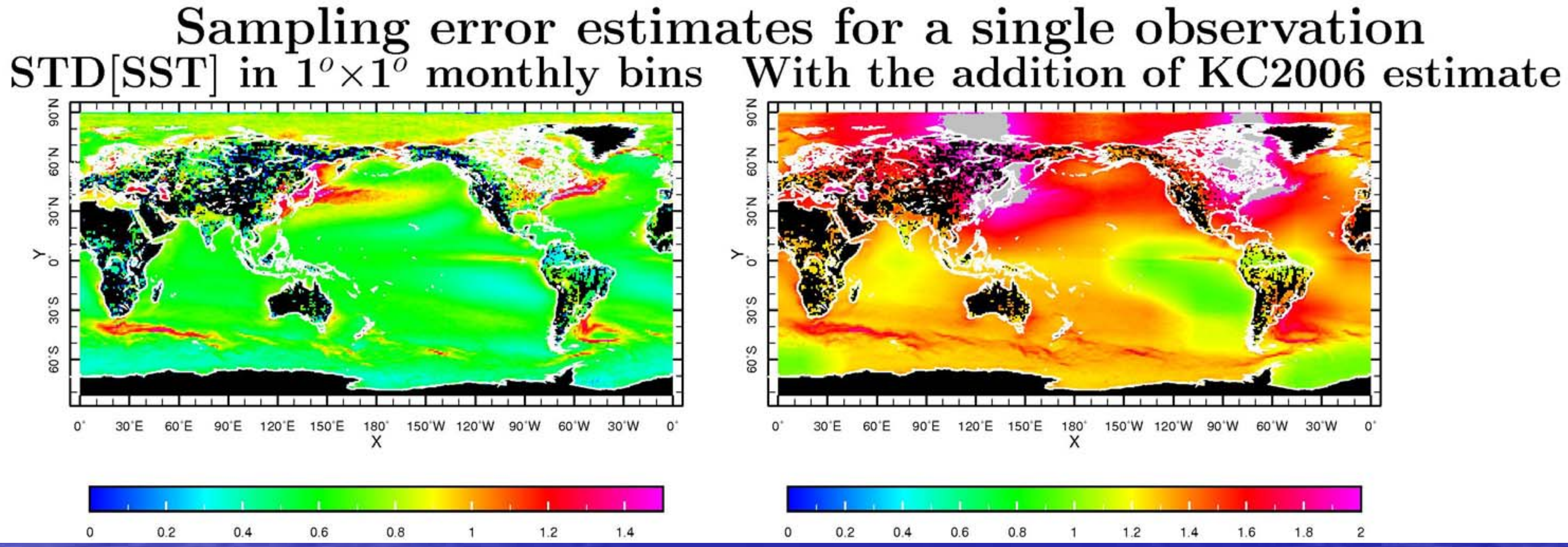
STD[SST] in ICOADS $1^o \times 1^o$ bins



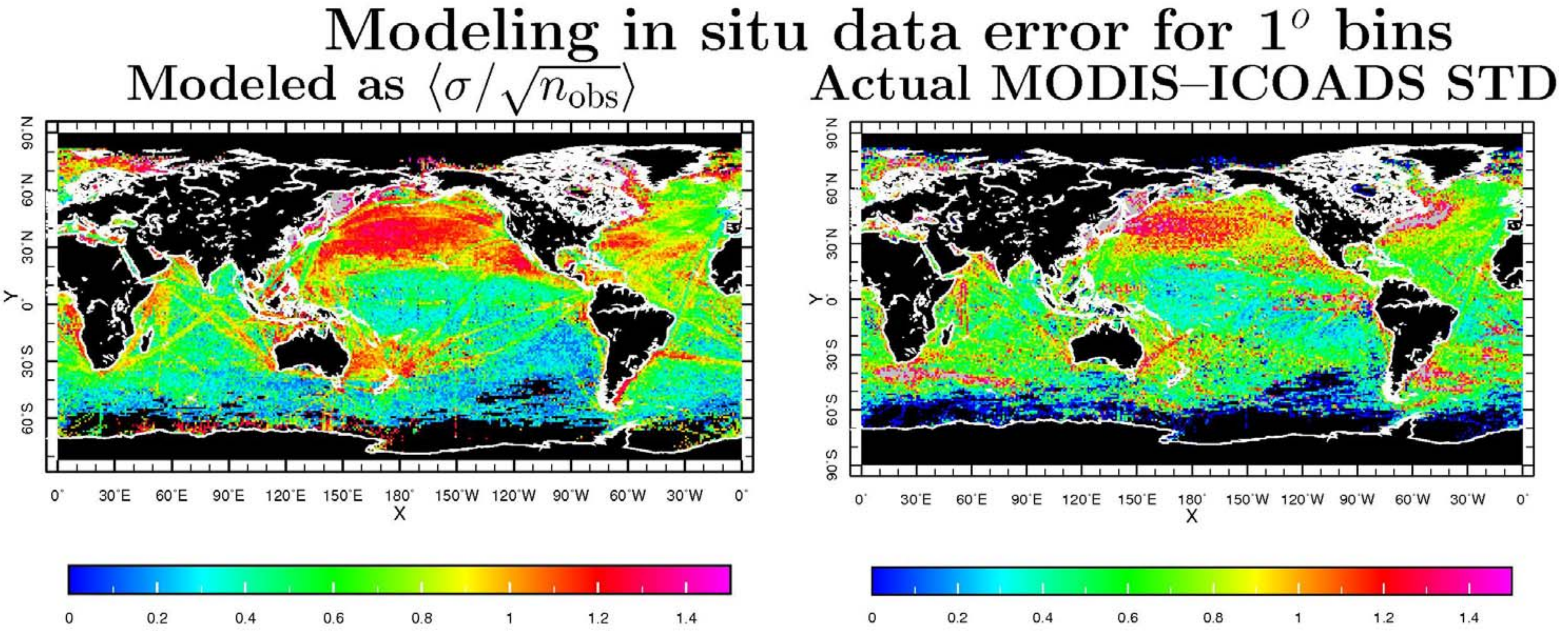
Measurement error (or very small-scale variability) has to be taken into account



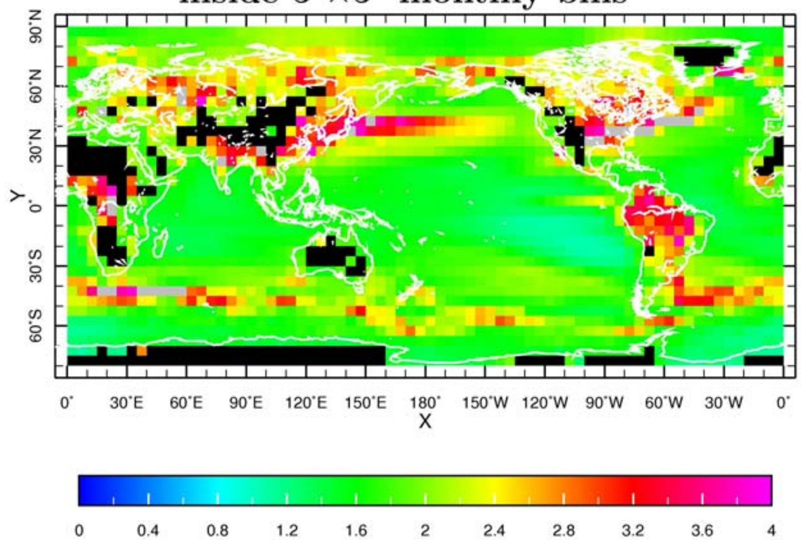
Combining the two estimates to obtain σ:



Does left look like right?

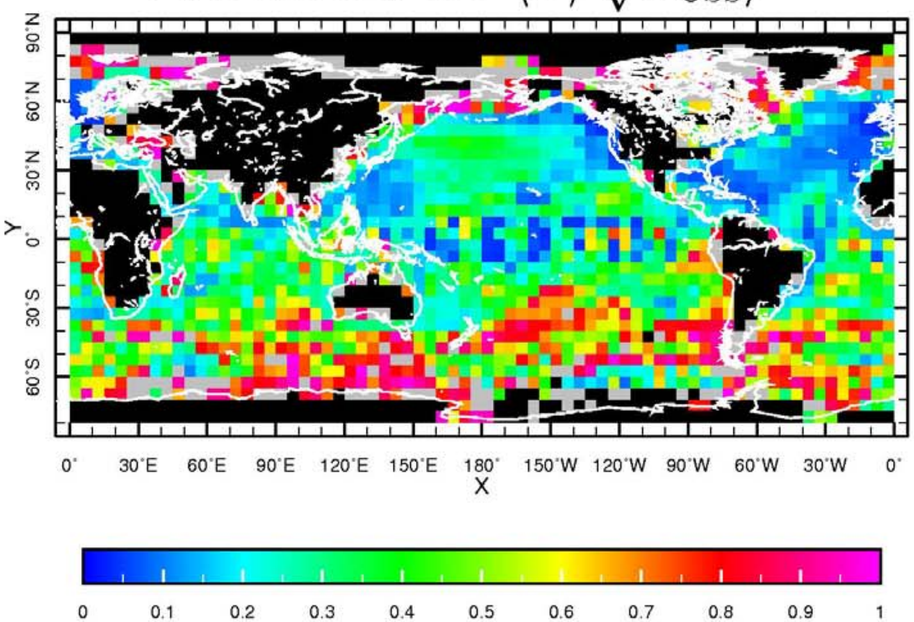


Single observation SST sampling+measurement error, ^{o}C, inside $5^{o}\times5^{o}$ monthly bins



Modeling in situ data error for 5^{o} bins

Modeled as $\langle \sigma/\sqrt{n_{\mathrm{obs}}} \rangle$



Actual MODIS–ICOADS STD