Data assimilation workshop

3D-Var exercise

Prepared by Junjie Liu

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Prologue
data assimilation package setup

=>cp ~temp/.cshrc ~/

=>source ~/.cshrc

=>cp -r /data/system4/jjliu/workshop_2007   ~/

=>cp -r /data/system4/jjliu/DAS_result
/data/system4/$USER

=>cp -r /data/system4/jjliu/nmcstat2_ini
/data/system4/$USER/nmcstat2

Note: $USER is your own user name

What will we learn in this exercise

- 3D-Var implementation formula
- How to construct the background error covariance
- NMC method
- The characteristics of the error structure
- Response test with 3D-Var (assimilating only one obs.)
- Run 3D-Var of rawinsonde observation network

3D-Var implementation formula

Cost function:

$$J(\delta\mathbf{x}) = \frac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}(\mathbf{H}\delta\mathbf{x} - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\delta\mathbf{x} - \mathbf{d})$$

$$\delta\mathbf{x} = \mathbf{x} - \mathbf{x}^b \qquad \mathbf{d} = \mathbf{y} - H(\mathbf{x}^b)$$

Background error covariance is too large to get the inverse, so we define a variable transformation (Barker et al., 2004):

$$\delta\mathbf{x} = \mathbf{U}\delta\mathbf{v}$$

$$J(\delta\mathbf{v}) = \frac{1}{2}\delta\mathbf{v}^T\mathbf{U}^T\mathbf{B}^{-1}\mathbf{U}\delta\mathbf{v} + \frac{1}{2}(\mathbf{H}\mathbf{U}\delta\mathbf{v} - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\mathbf{U}\delta\mathbf{v} - \mathbf{d})$$

3D-Var implementation (continued)

In order to make the covariance matrix to be the identity:

$$\mathbf{B} = \mathbf{U}\mathbf{U}^T$$

$$J(\delta\mathbf{v}) = \frac{1}{2}\delta\mathbf{v}^T\delta\mathbf{v} + \frac{1}{2}(\mathbf{H}\mathbf{U}\delta\mathbf{v} - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\mathbf{U}\delta\mathbf{v} - \mathbf{d})$$

The gradient of the cost function is:

$$\nabla J(\delta\mathbf{v}) = \delta\mathbf{v} + \mathbf{U}^T\mathbf{H}^T\mathbf{R}^{-1}(\mathbf{H}\mathbf{U}\delta\mathbf{v} - \mathbf{d})$$

Using cost function and the gradient of the cost function, we use a quasi-Newton minimizer to find the solution $\delta\mathbf{v}$, and then convert $\delta\mathbf{v}$ to $\delta\mathbf{x}$

Quasi-Newton Minimizer

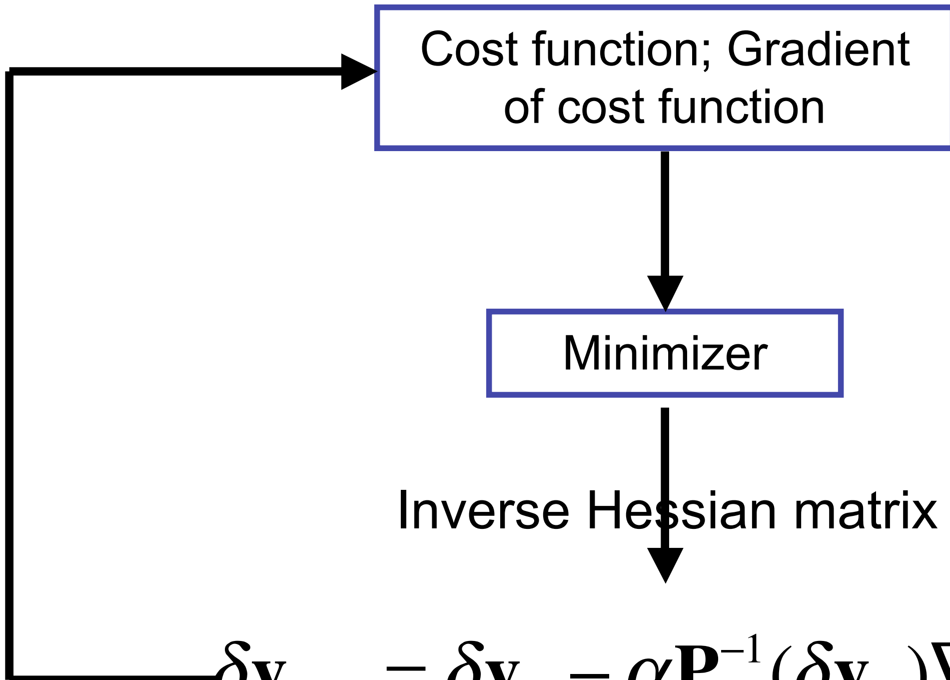
Quasi-Newton's method:

$$\delta \mathbf{v}_{k+1} = \delta \mathbf{v}_k - \alpha \mathbf{P}^{-1}(\delta \mathbf{v}_k)\nabla J(\delta \mathbf{v}_k)$$

Central idea underlying quasi-Newton method is to use an approximation of the inverse Hessian $\mathbf{P}^{-1}$

Inverse Hessian matrix here is the second derivative of the cost function, which is the analysis error covariance in the data assimilation framework

Minimizing process



$$\delta \mathbf{v}_{k+1} = \delta \mathbf{v}_k - \alpha \mathbf{P}^{-1}(\delta \mathbf{v}_k)\nabla J(\delta \mathbf{v}_k)$$

Construction of the variable transformation $\mathbf{U}$

$$\delta\mathbf{x} = \mathbf{U}\delta\mathbf{v} \qquad \mathbf{B} = \mathbf{U}\mathbf{U}^T$$

The essential problem is to construct the variable transformation:

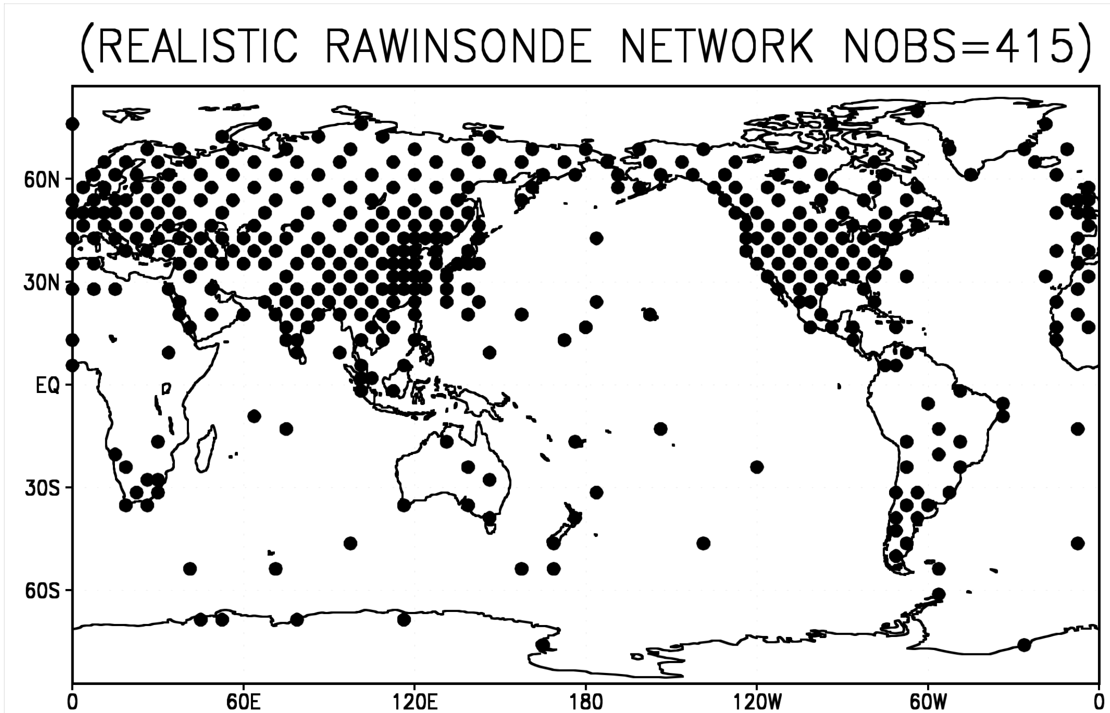
$$\mathbf{U} = \mathbf{VCA}$$

A: error standard deviation

C: spatial error correlation, which includes horizontal and vertical correlation. Assume Gaussian shape in the horizontal, so it only needs to store the length scale

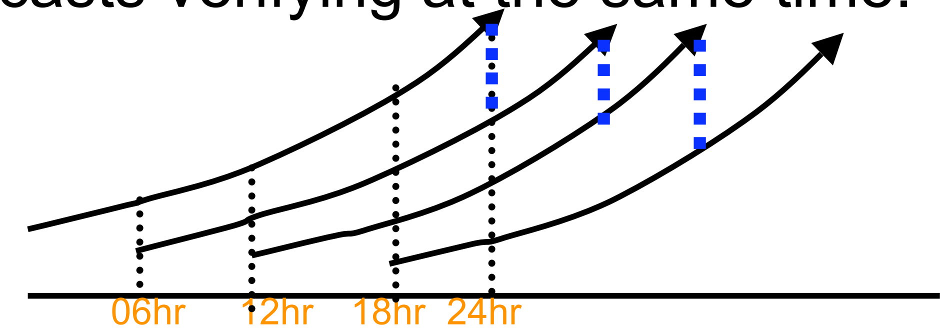
V: inter-variable error correlation, which only considers geostrophic balance.

Construct background error covariance based on rawinsonde observation network



Creation of the transformation matrix U based on NMC method (Parrish and Derber, 1992)

The structure of the forecast error covariance is estimated as the average over many differences between two short-range model forecasts verifying at the same time.



1. The blue dashed lines represent the difference between 18hr forecast and 24hr forecast.
2. Based on these forecast differences, we can calculate all the statistics (A, C, V) need to construct the background error covariance B

Code to create the 18hr and 24hr forecasts

1. Enter to the folder

=>cd ~/workshop_2007/DAS/tdvar_stat

2. Main program: tdvar.f90, tdvar_tools.f90

3. Run tdvar_nmc.sh

(use one of the editors to change the ‘USER=jjliu’ to your own user name in ‘tdvar_nmc.sh’)

=>./tdvar_nmc.sh

Code to calculate A, C, V

$$\mathbf{B} = \mathbf{U}\mathbf{U}^T \qquad \mathbf{U} = \mathbf{VCA}$$

1. The main program: nmc_stat.f90, which calculates the correlation, and inter-variable correlations as well.

2. Run nmc_stat.sh

=>cd ~/workshop_2007/DAS/tdvar_stat

(use one of the editors to change the 'USER=jjliu' to your own user name)

=>./nmc_stat.sh

3. The result is in the folder: ~/workshop_2007/DAS/tdvar_stat/dat_stat

Exercises: Background error structure

Make plots of the results (use grads commands):

=>cd ~/workshop_2007/tdvar_stat/dat_stat

=>grads64

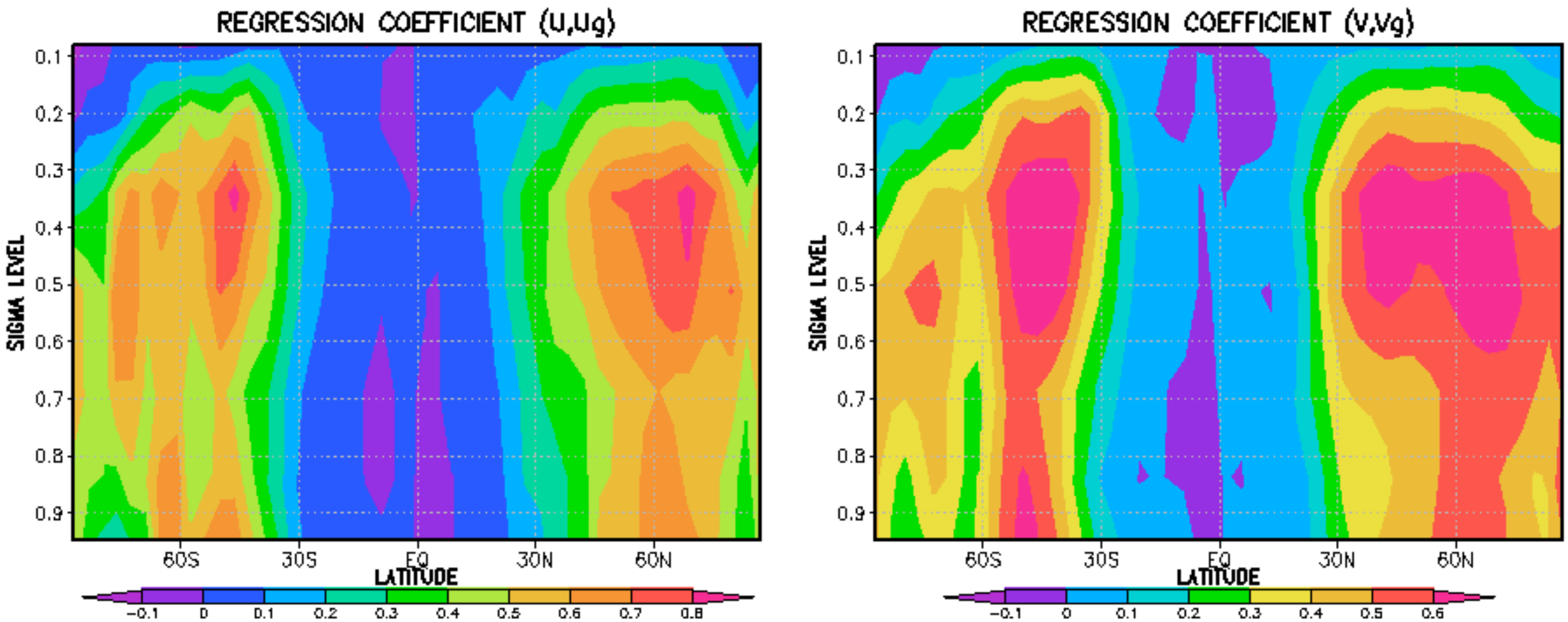
=>reg.gs → The regression coefficient between v and geostrophic wind background errors

=>xcorr.gs

=>ycorr.gs → Correlation of the background errors in the x and y direction

=>stdev.gs → Background error standard deviation

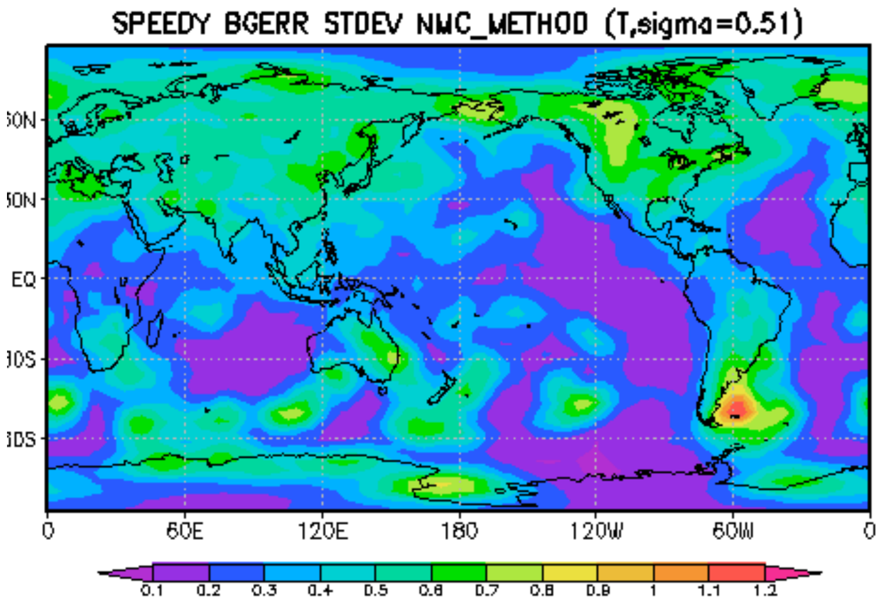
Inter-variable background error relationship (geostrophic balance)

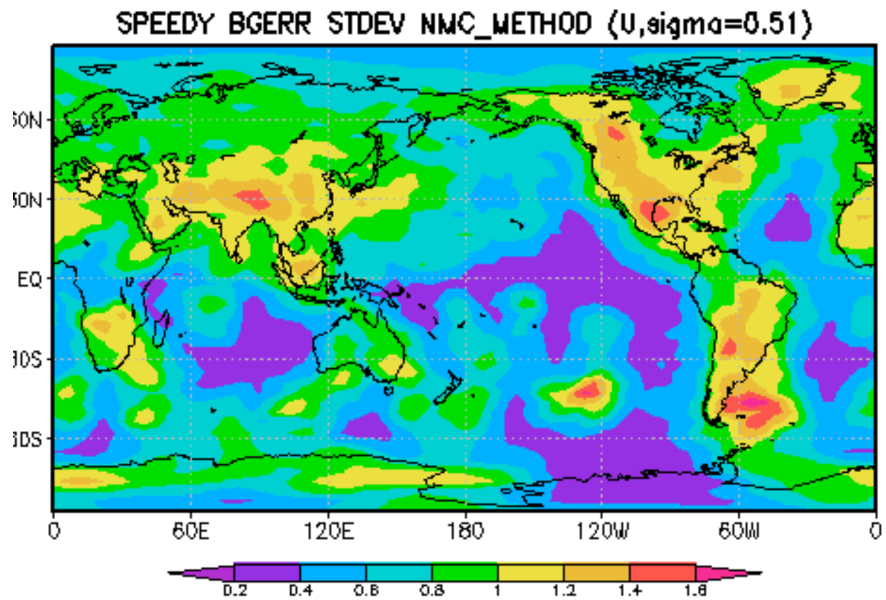


Geostrophic error balance concentrates over mid-latitudes.

Almost no geostrophic wind error correlation over Tropics

3D-Var estimated background error standard deviation

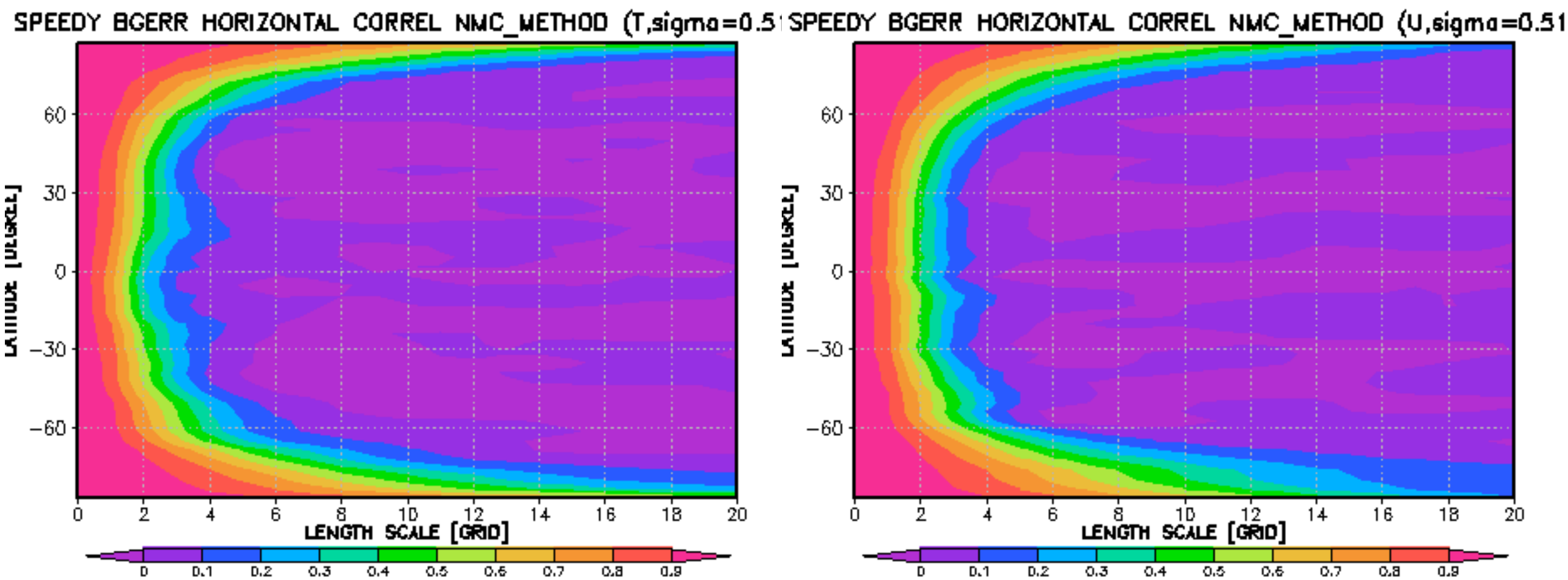




Strong spatial dependence of the background error standard deviation for both temperature (left panel) and zonal wind (right panel).

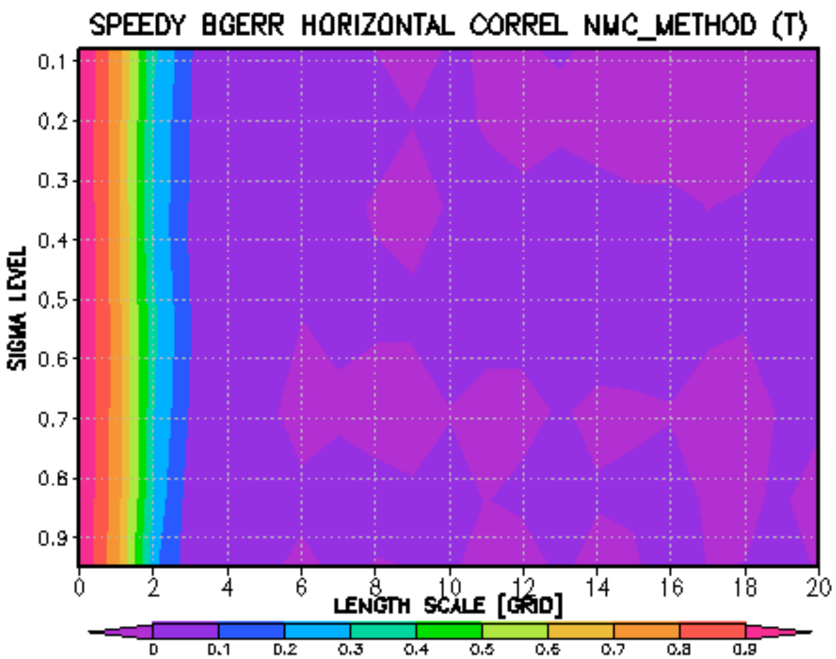
Large error standard deviation over land, since rawinsonde concentrates over land.

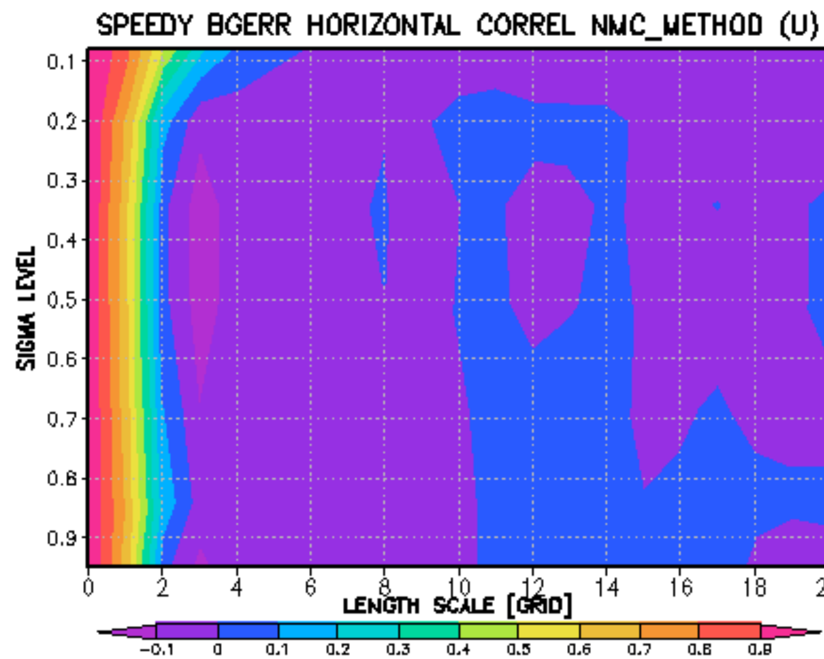
Zonal error correlation between adjacent grid points



Error correlation between adjacent points increases with latitude because of the convergence of the meridians

meridional error correlation



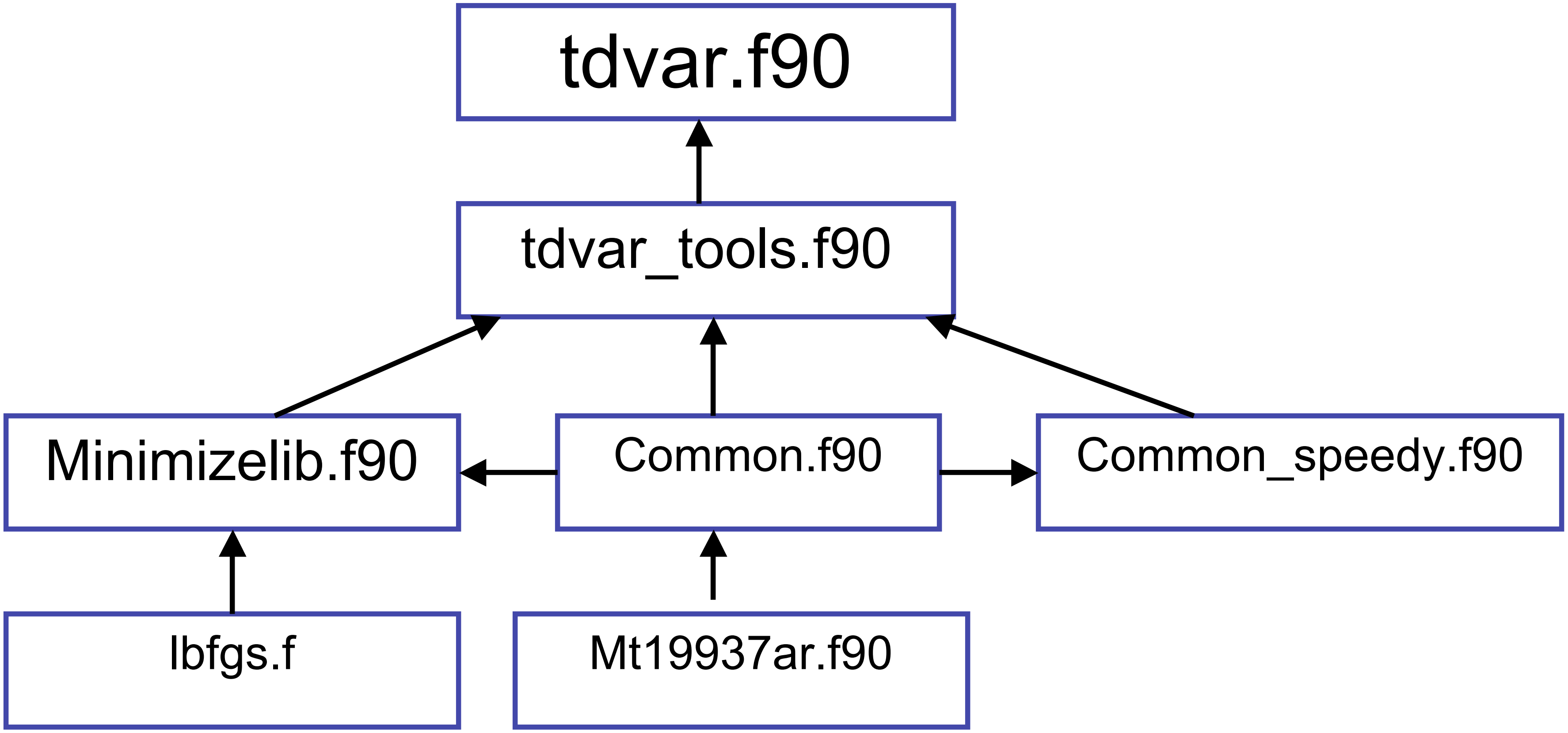


Almost has no dependence on height

The correlation decreases to zero beyond the second grid points

Code structure

~/workshop_2007/DAS/tdvar



3D-Var response test

In this experiment, we only assimilate one observation to see how the analysis works. (The observation location is (76, 35, 4), and it is zonal wind)

1. Run tdvar_response.sh (~/workshop_2007/DAS/tdvar)

=>cd ~/workshop_2007/DAS/tdvar

(Change 'USER=jjliu' to your own user name in 'tdvar_response.sh')

=>./tdvar_response.sh

2. View the result and answer the following question:

   What is the analysis increment structure?

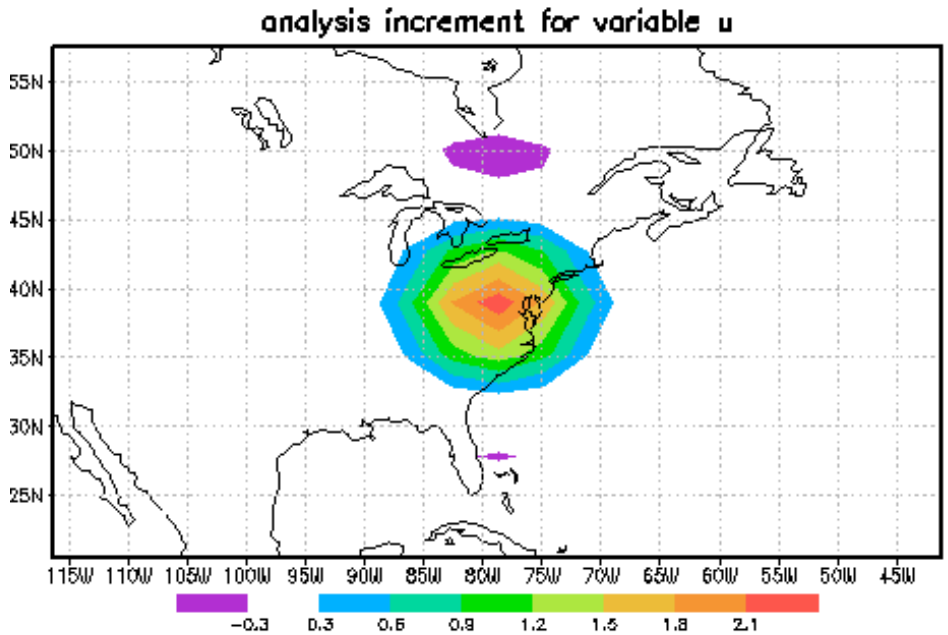
=> cd /data/system4/$USER/DAS_result/3dvar/response/gs

=>grads64

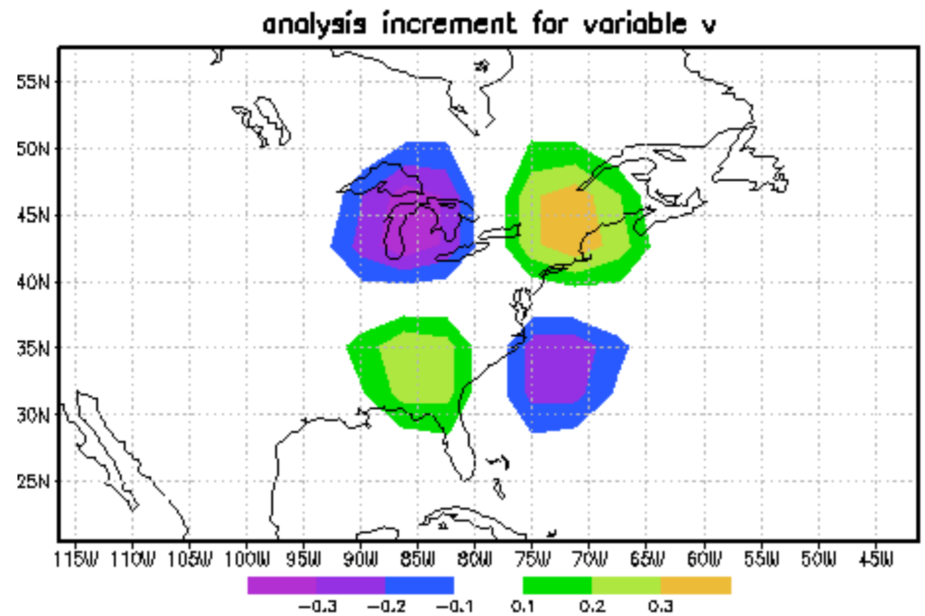
=>anal_incre.gs

Analysis increments

Zonal wind



Meridional wind

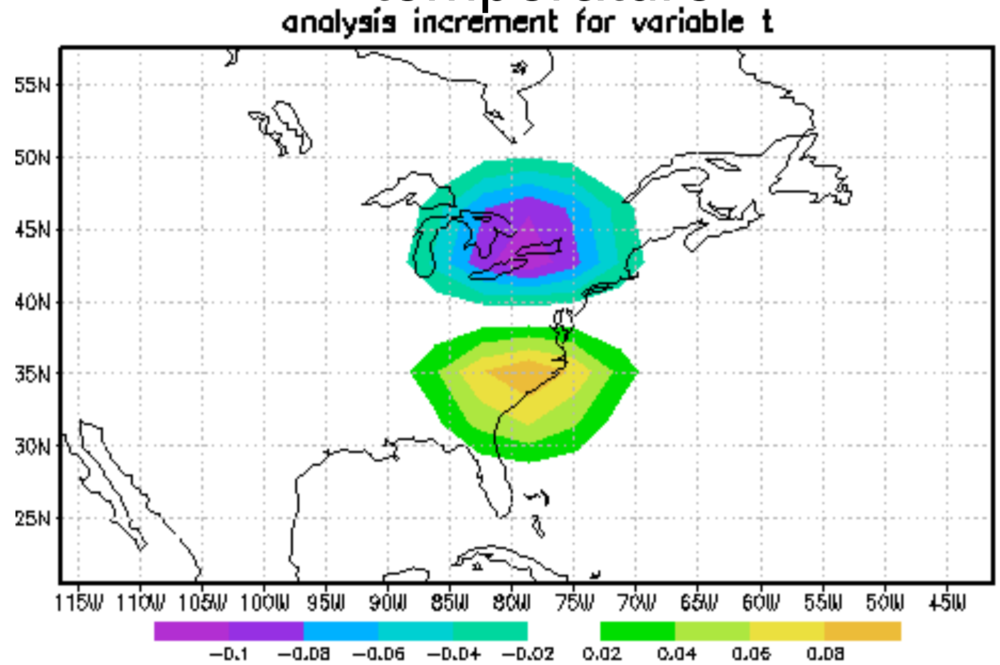


The positive zonal wind u increment leads to negative v increment in the north-west---south-east quadrants

Compare with Kalnay (2003, pp163)

Analysis increment

temperature

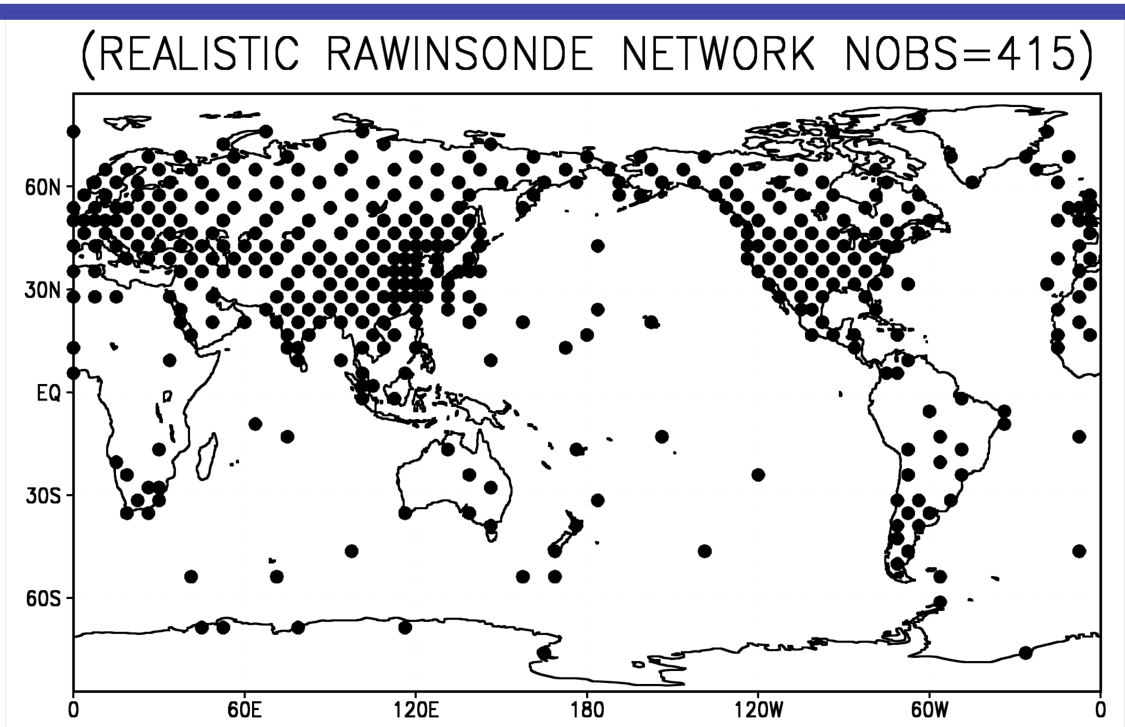


The positive zonal wind increment leads to negative temperature increment in the north. Kalnay (2003, pp163)

Exercise

- Read Kalnay (2003) pp163.
- If you only observe only meridional wind at one point, what is the analysis increment of other variables?
- Note: observation location is controlled in ex_obs_response.f90

Run 3D-Var of rawinsonde observation network



- =>cd ~/workshop_2007/DAS/tdvar
- Change the ‘USER=jjliu’ to your own username in ‘tdvar.sh’ file
- =>./tdvar.sh

Note: with different observation network, the parameter ‘stdfact’ in ‘tdvar_tools.f90’ has to be tuned.

Review the results and answer the questions

Question:

how does the analysis error converge? And how long does it take?

=>cd /data/system4/$USER/DAS_result/3dvar/EXP/gs

=>grads64

=>RMS_err_500hPa_u.gs

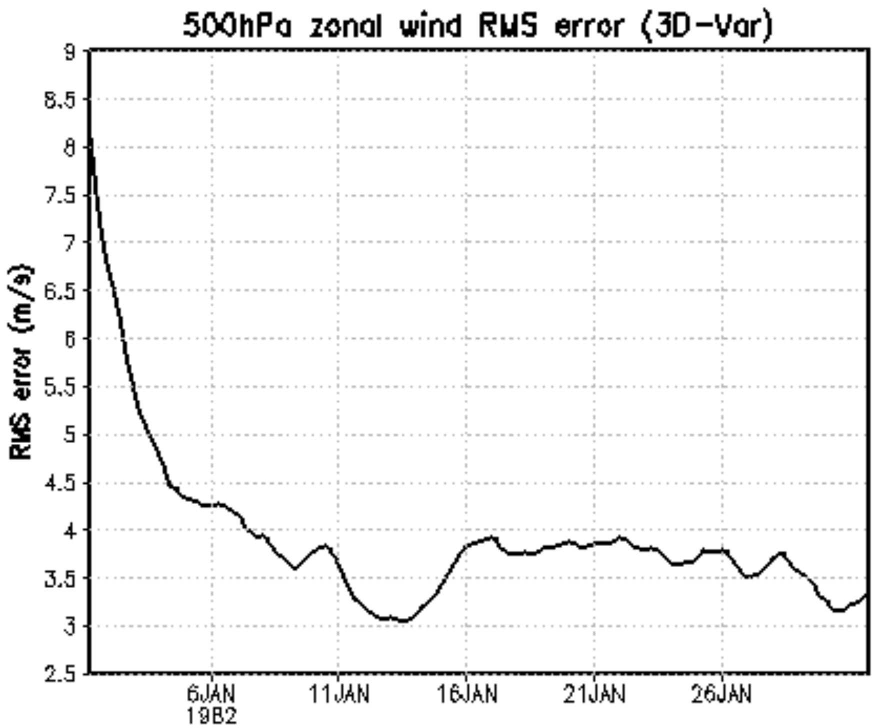
How does the zonally-averaged analysis error structure look like? And why?

=>cd /data/system4/$USER/DAS_result/3dvar/EXP/gs

=>grads64

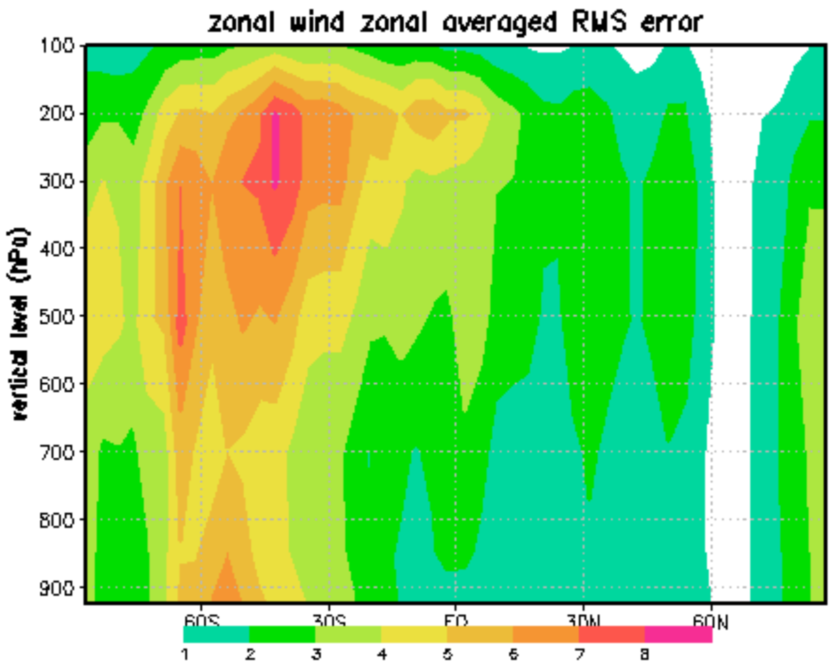
=>RMS_zonal_u.gs

500hPa zonal wind RMS error



Why the error is still bigger than the observation error (1m/s)?

Zonal averaged zonal wind RMS error



Why are the errors larger in the SH?

References

- Barker, D. M., W. Huang, Y.-R. Guo, A. J. Bourgeois, and Q. N. Xiao, 2004: A Three-Dimensional Variational Data assimilation system for MM5: Implementation and initial results. *Mon. Wea. Rev*., **132**, 897-914.

- Kalnay, E., 2003: Atmospheric modeling, data assimilation and predictability. Cambridge University Press, 341pp

- Miyoshi,T., 2005: Ensemble Kalman filter experiments with a primitive-equation global model, thesis in University of Maryland

- Parrish, D. F. and J. C. Derber, 1992: The National Meteorological Center's Spectral Statistical-Interpolation Analysis system. *Mon. Wea. Rev*. **120,** 1747-1763