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Satellite observations of clouds[1]: overview

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(1: with an eye toward assimilation)

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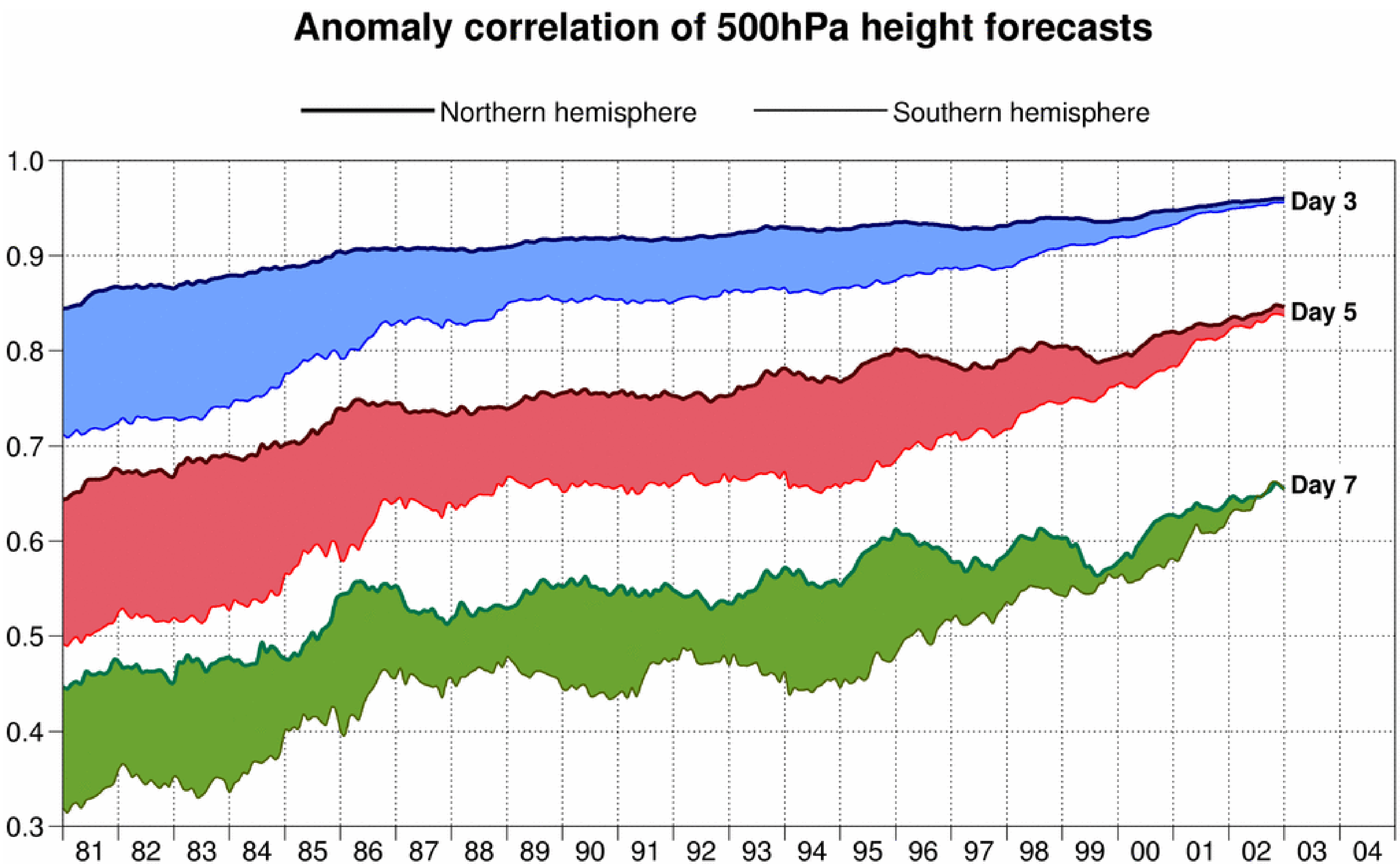
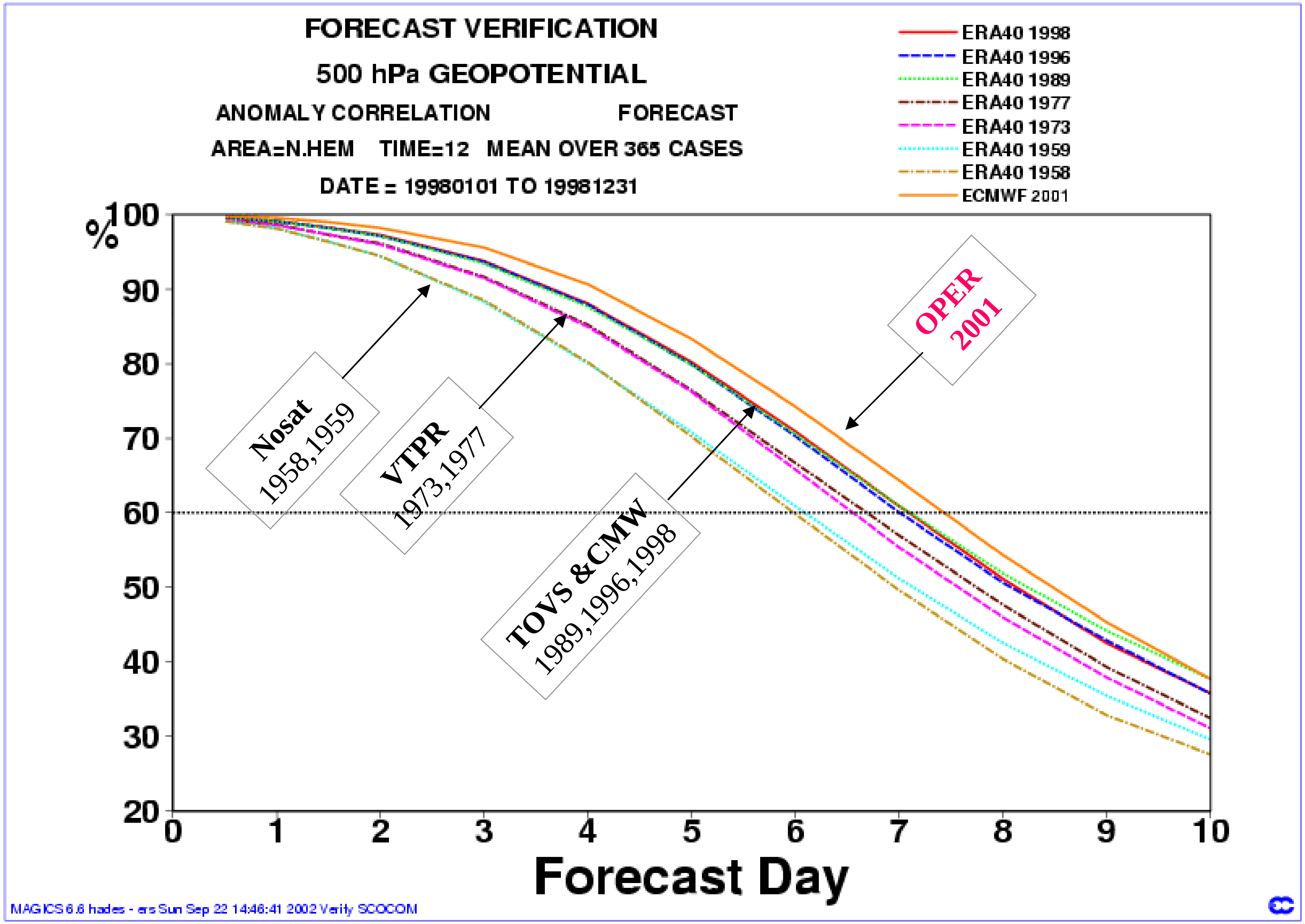
Outline

1. Context
2. Overview
3. A new dimension
4. Model 'verification'
5. Summary

Influence of satellite observations on forecast skill for NH and SH





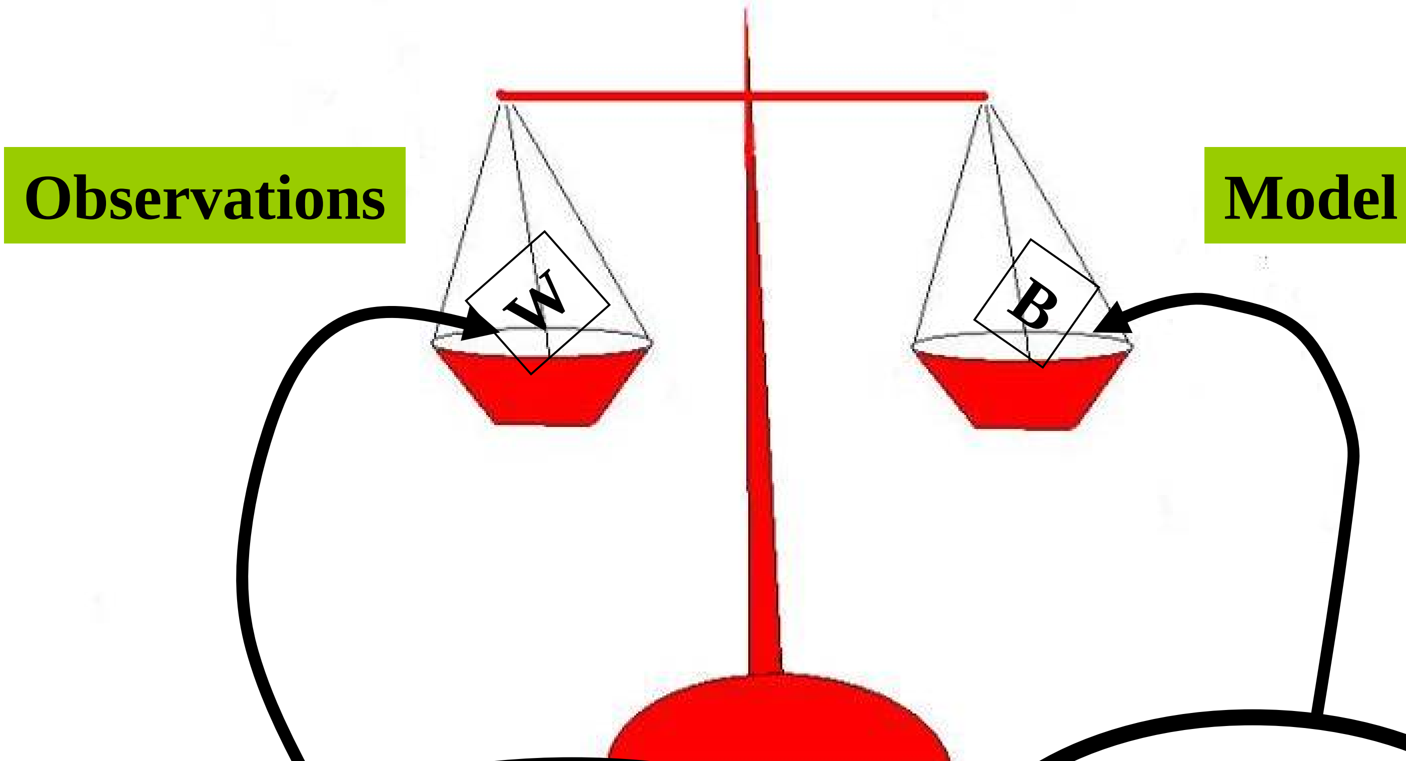
FORECAST VERIFICATION
500 hPa GEOPOTENTIAL
ANOMALY CORRELATION FORECAST
AREA=N.HEM TIME=12 MEAN OVER 365 CASES
DATE = 19980101 TO 19981231
ERA40 1998
ERA40 1996
ERA40 1989
ERA40 1977
ERA40 1973
ERA40 1959
ERA40 1958
ECMWF 2001
%
100
90
80
70
60
50
40
30
20
0
1
2
3
4
5
6
7
8
9
10
Forecast Day
Nosat 1958,1959
VTPR 1973,1977
TOVS &CMW 1989,1996,1998
OPER 2001
MAGICS 6.6 hades - ers Sun Sep 22 14:46:41 2002 Verify SCOCOM

Assimilation of ‘moist physics’ observations

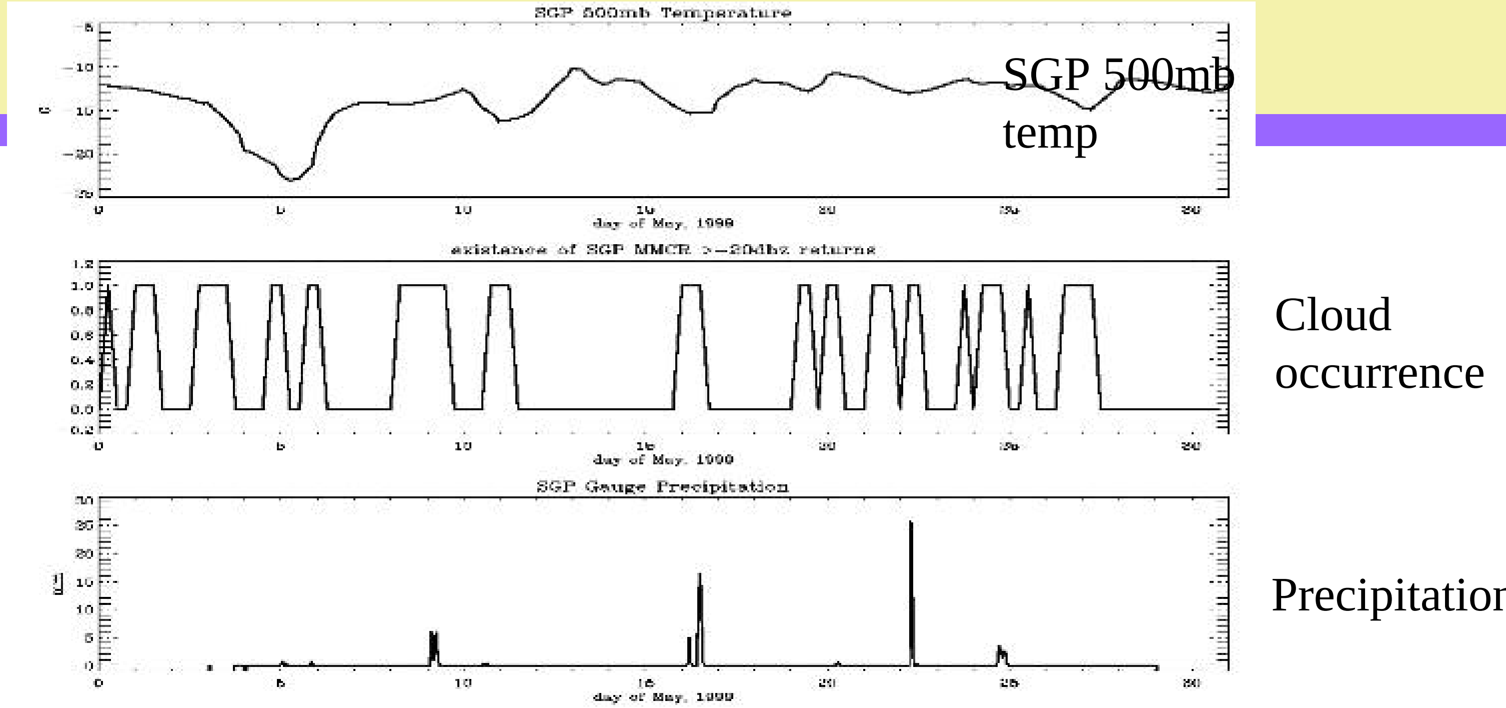
1. Obvious importance of clouds and precipitation
   Satellite data represent 95% of the data ingested into the ECMWF analysis system, but most of the satellite radiances (about 75 %) are discarded because they are diagnosed as cloud- or rain-affected.

2. Assimilation of moist variables into NWP is challenging due to the wide range of spatial and temporal scales of (non-linear) moist processes and lack of *real* model error assigned to them

Retrieval & ‘assimilation’ are essentially the same problem



$$J = \frac{1}{2}(P(x)-y)^T W^{-1}(P(x)-y) + \frac{1}{2}(x_b - x)^T B^{-1}(x_b - x)$$



$$\Phi = [x-x_a]^T B^{-1} [x-x_a] + [y-f(x)]^T S_y^{-1} [y-f(x)]$$

Seek x, such that $d\Phi/dx \rightarrow 0$

X_a = background

B^{-1} = prediction (model) error

$f(x)$ = model of observation

$S_y^{-1}(W^{-1})$ = 'observation' error

1. How? Linear Physics?
2. What? Prognosed variables, time & space filtered?? What statistics?
3. Forecast model error? How is this defined?
4. Improving & understanding observing systems

Steps Toward a strategy for operational assimilation of cloud and precipitation obs:

- Optimizing the choice of observations [y(t)]

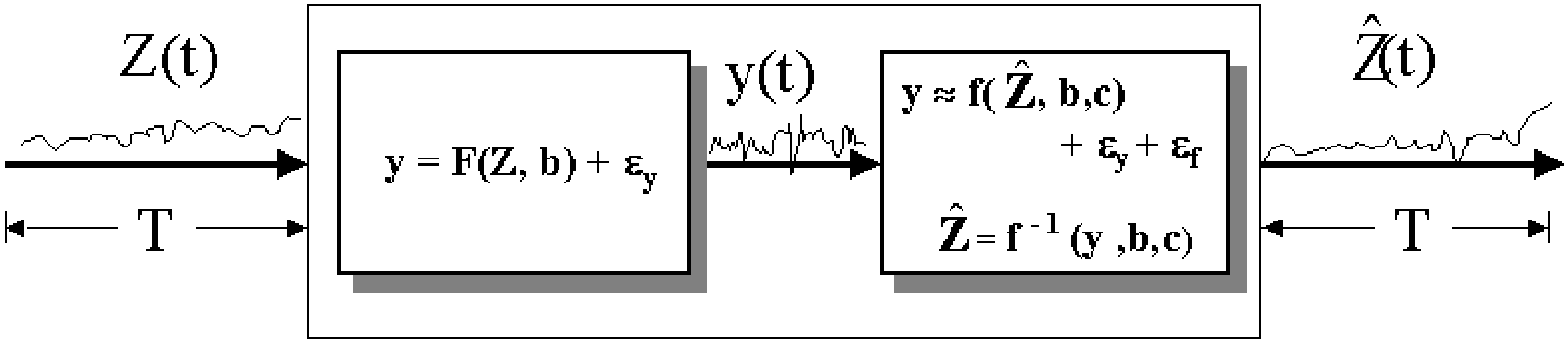
- Model evaluation using current and new satellite measurements [B^{-1}]

- Development of new and improved 'moist physics' (clouds and especially convection) [B^{-1}]

- Develop, test and quantify errors of 'observational operators associated with moist physics observations' (i.e. IR, solar and microwave radiative transfer schemes for clouds & precip, radar reflectivity models, etc) [f(x) & W^{-1}]

- Research on the optimal strategy to assimilation (e.g tangent linear, ensemble methods etc…) [i.e. $d\Phi/dx \rightarrow 0$]

A satellite ‘Observing System’



Two key components of the ‘transfer function’ – the forward and inverse functions

Measurements y(t) are connected to the ‘state’ Z

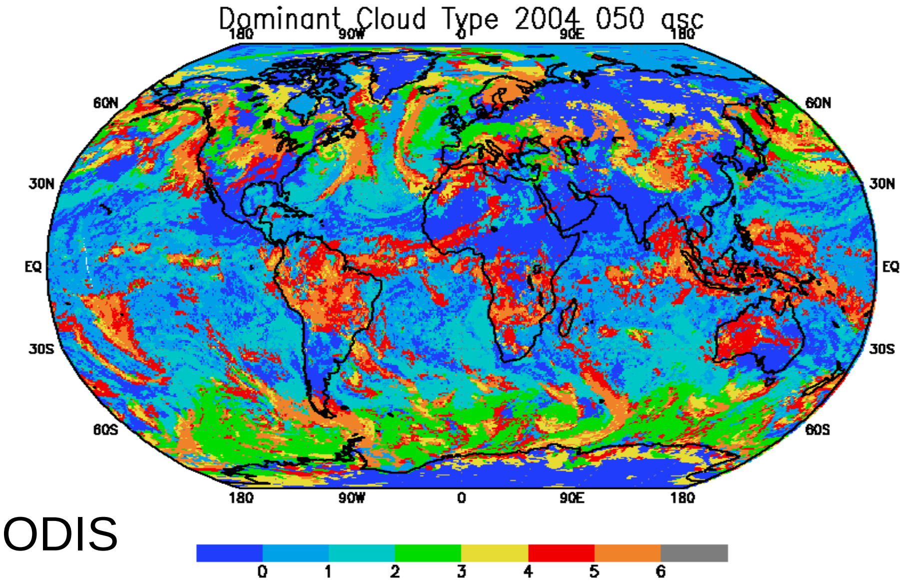
The state is inferred (retrieved) given the measurement, a physical model and other ‘knowledge’ about the system.

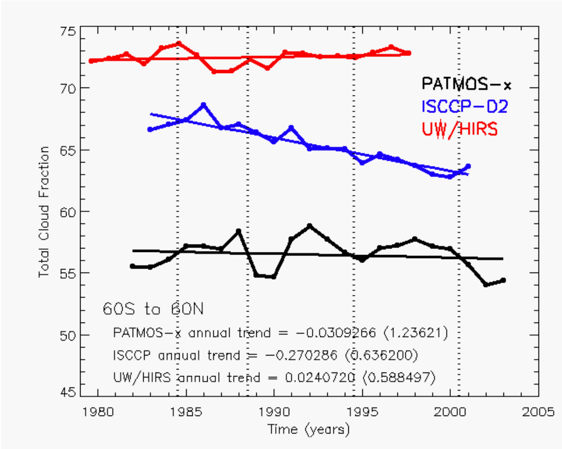
Key parameters & ‘knowledge’:

- Measurement, y(t) and error ε_y
- Model f & its error ε_f
- Model parameter b
- Constraint parameters c

Cloud occurrence (e.g. PATMOS, ISCCP, HIRS, MODIS etc)

PATMOS



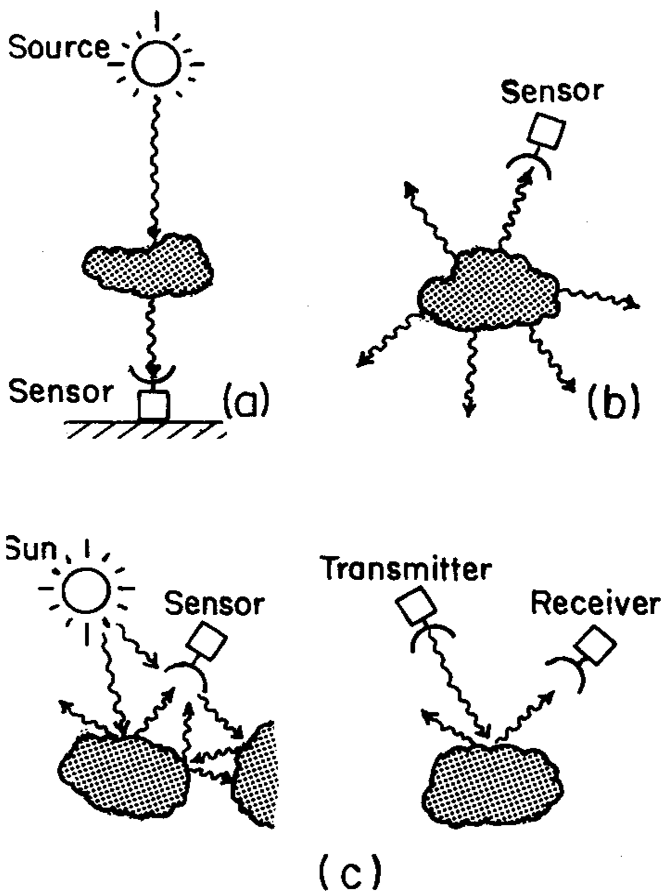


Decadal cloud amount trends, precipitation variability

MODIS

Physical basis for satellite observations of cloud properties (ie different types of f(x)’s)

extinction emission



scattering

Passive (radiometry)

These methods provide primarily *path integrated* information – i.e. little or no vertical structure:

Examples considered – scattered sunlight and cloud ‘optical’ properties, thermal emission and microwave emission

Active (lidar, radar and mm $\rightarrow$ cm wavelengths)

Profile information about occurrence, optical properties, microphysics and bulk water mass – example highlighted is of mm-wave radar

One of the messages conveyed in this overview

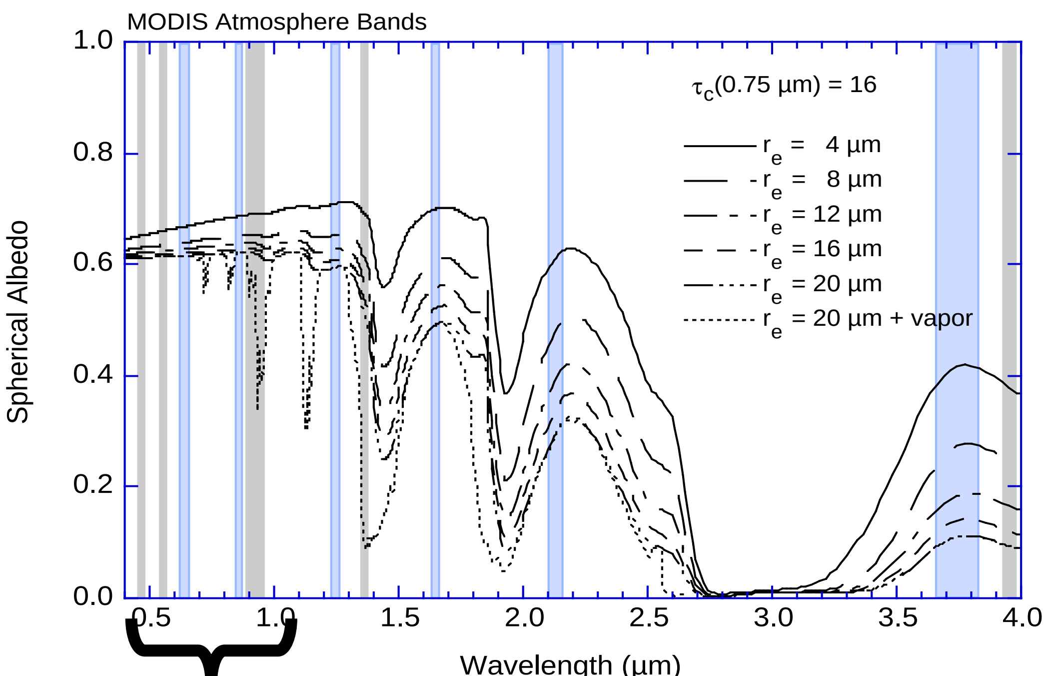
Most cloud & precipitation retrievals are single sensor & ‘physics’ centric – leaving us to ponder which of the seemingly myriad of different approaches is optimal, how accurate is the retrieved information and what is to be gained in combining different types of measurements ?

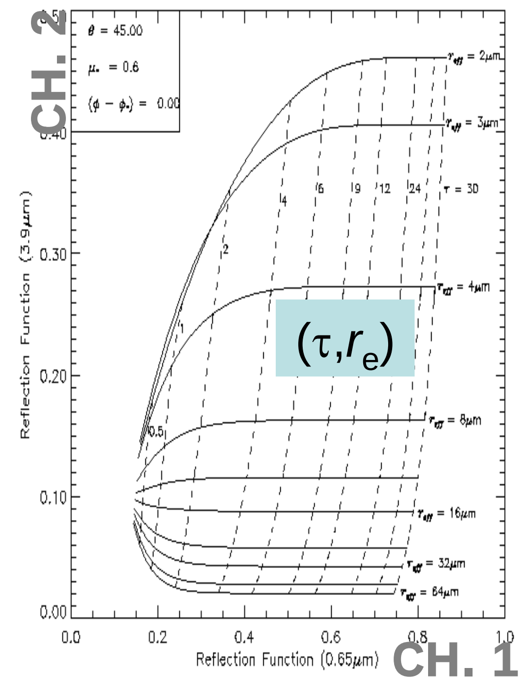
The future is perhaps with multi-sensor ‘assimilation ‘ of information as, for example, exemplified by the upcoming A-Train

Cloud optics and 'microphysics' : solar scattering

Reflectance variations at these wavelengths → optical depth and r_e variations

Optical properties



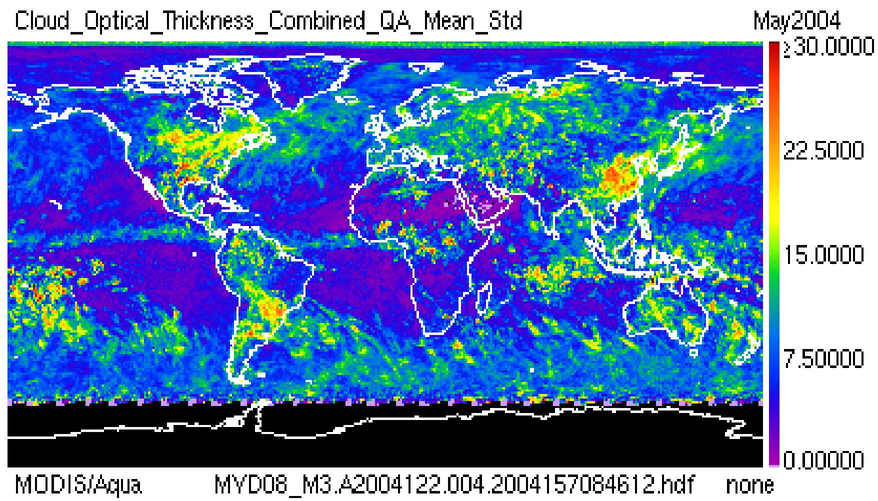


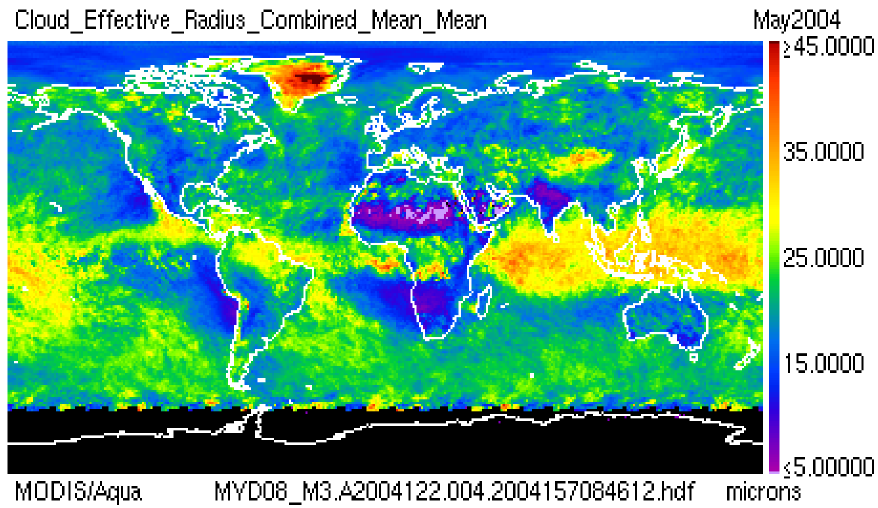
Reflectance variations at these wavelengths → optical depth variations

Twomey & Cocks, 1980's
Nakajima & King, 1990s

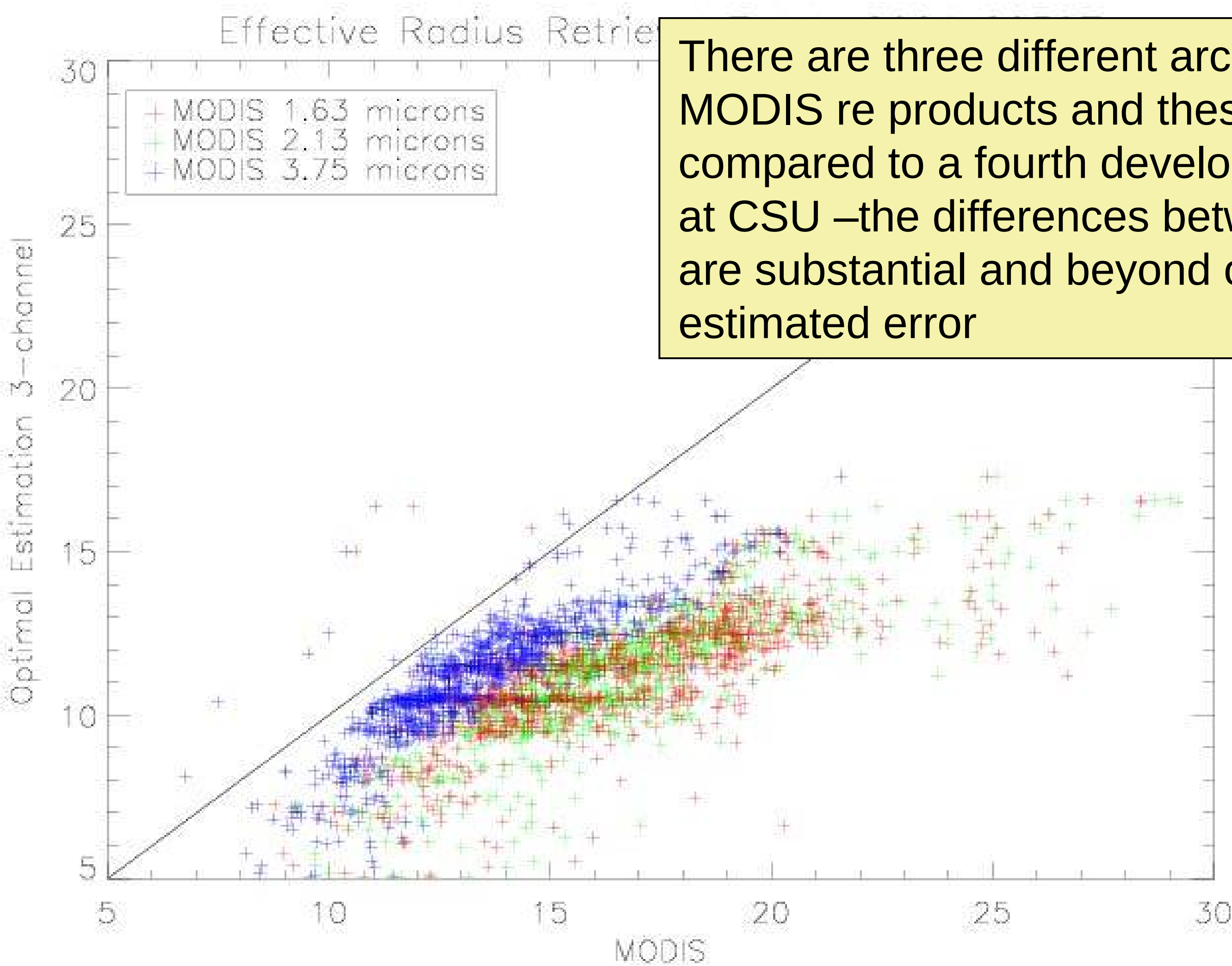
$$LWP \rightarrow \frac{2}{3}\tau r_e$$

An example: MODIS optical property information





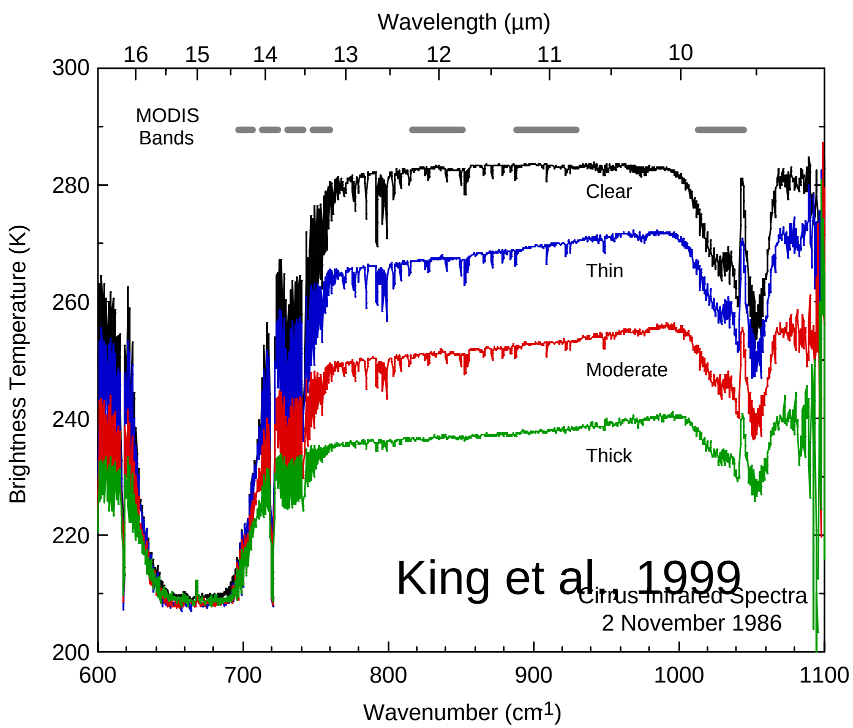
Particle Size retrieval examples – low level water clouds



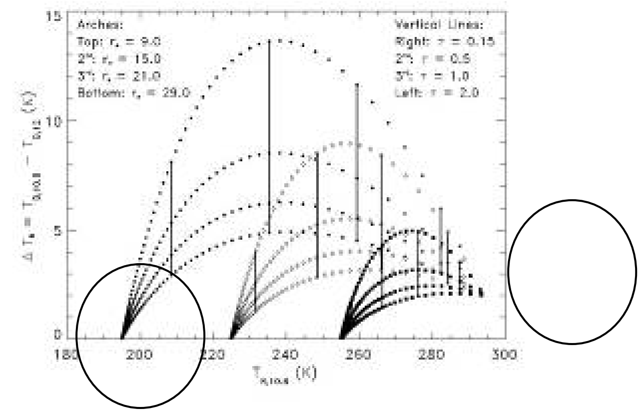
There are three different archived MODIS re products and these are compared to a fourth developed by us at CSU –the differences between them are substantial and beyond our estimated error

Split window thermal emission

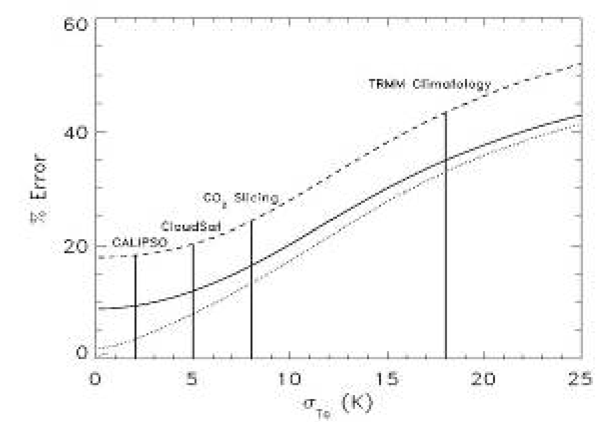
Optical properties



King et al., 1999



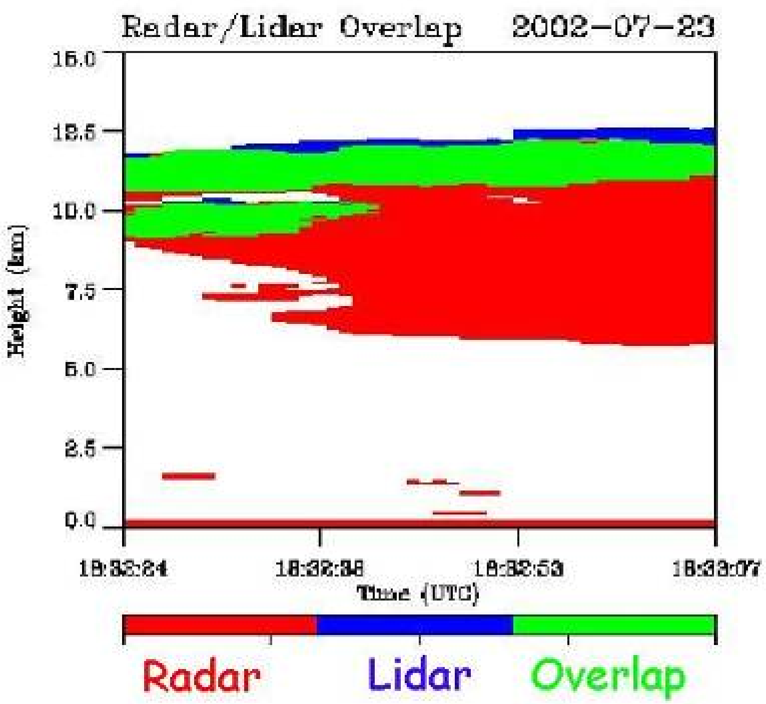
Given the 11 μm cloud emission and clear sky temperatures, then optical depth and re follow from ΔT_b and T_{11}.

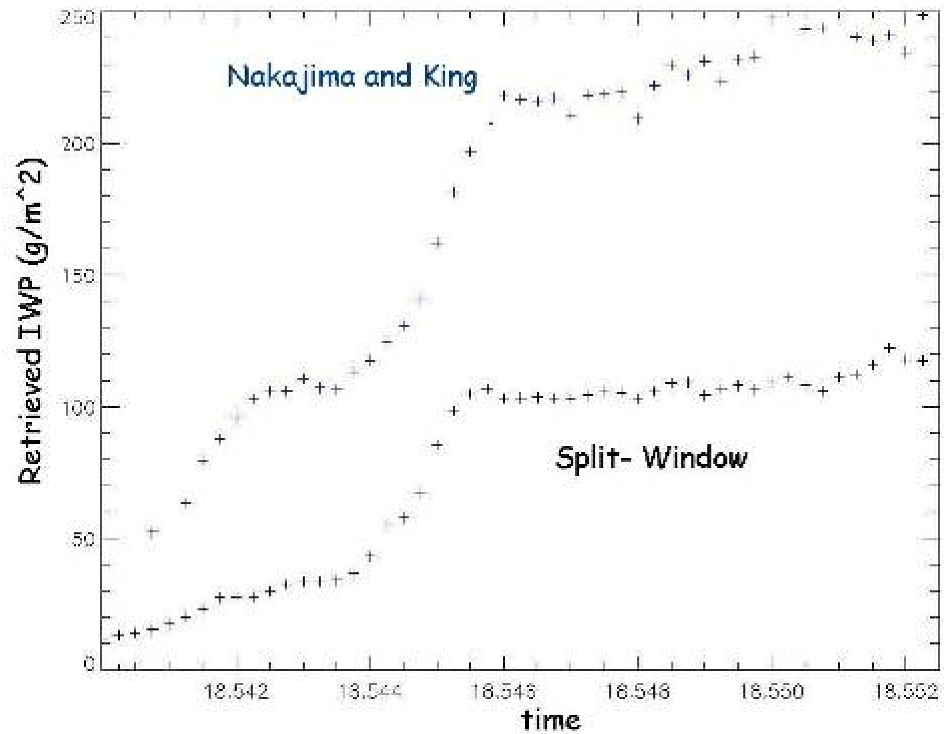


Cooper et al., 2003

'Same' optical information as scattering method but limited to (optically) thin clouds

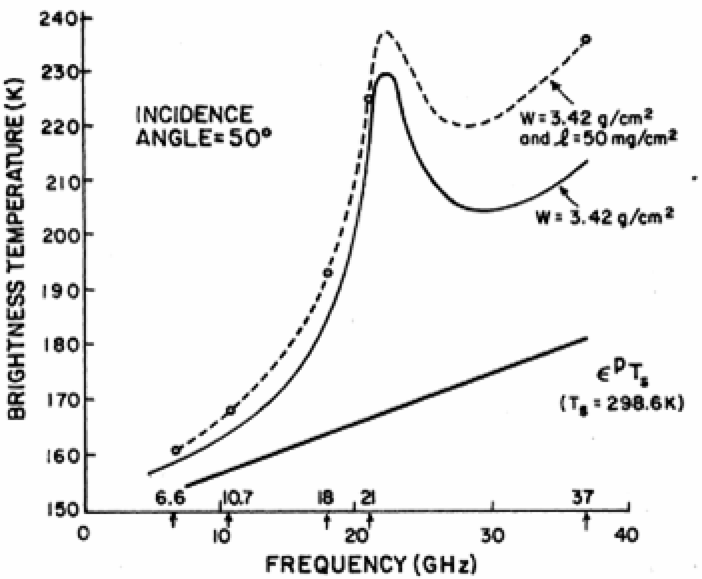
Cooper et al., 2005)





There is no real attempt to achieve a level of ‘consistency’ between different retrieval schemes even using measurements from the same instrument

Microwave emission –cloud liquid water path



Microwave spectrum around the 22 GHZ water vapor absorption line

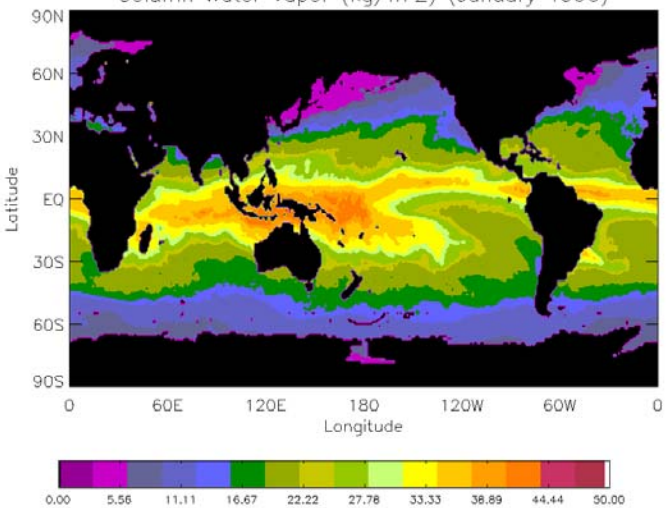
$$\left(e^{-\tau^*/\mu}\right)_w \sim e^{-k_w w}$$

$$\left(e^{-\tau^*/\mu}\right)_\ell \sim e^{-k_\ell W}$$

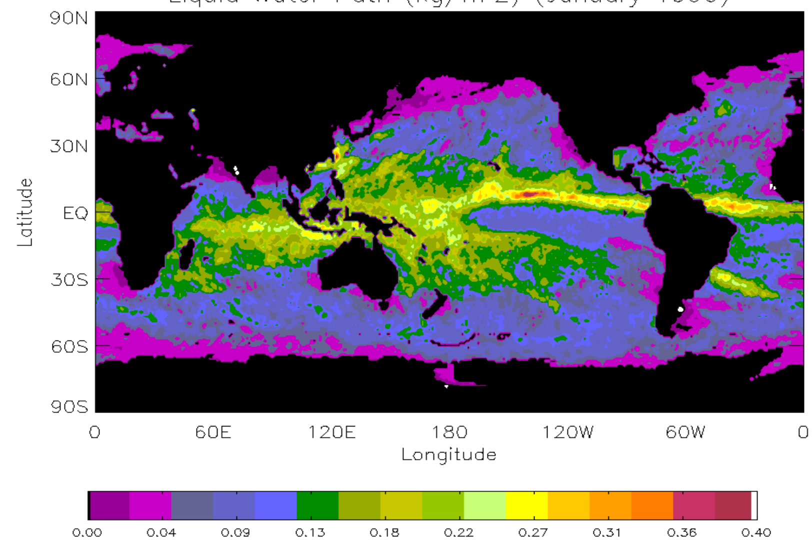
$$k_\ell W + k_w w = \frac{\mu}{2}\ln\frac{\Delta T}{T_S(R_{l/V} - R_{r/H})Tr_{ox}^2}$$

Measurement of ΔT at two frequencies (19GHz, 37 GHz), estimation of RV/H+ Δkw/l, and Trox allows for simultaneous solution for w and W,

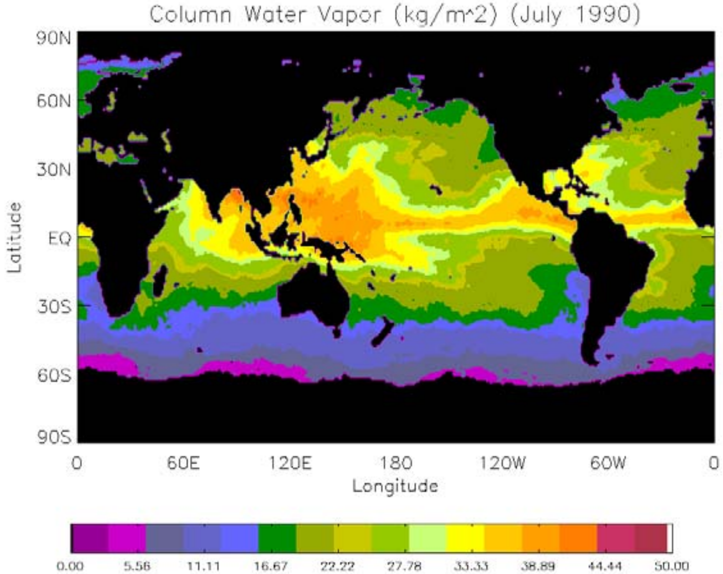
Column Water Vapor (kg/m^2) (January 1990)



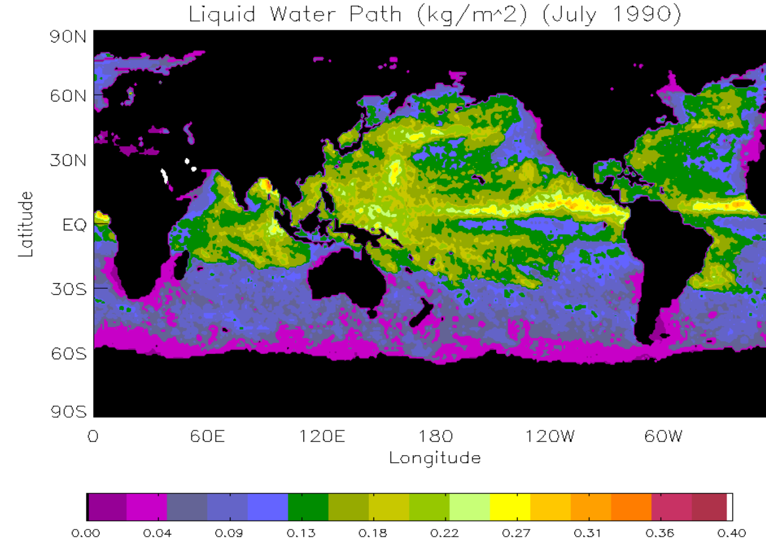
Liquid Water Path (kg/m^2) (January 1990)



Column Water Vapor (kg/m^2) (July 1990)

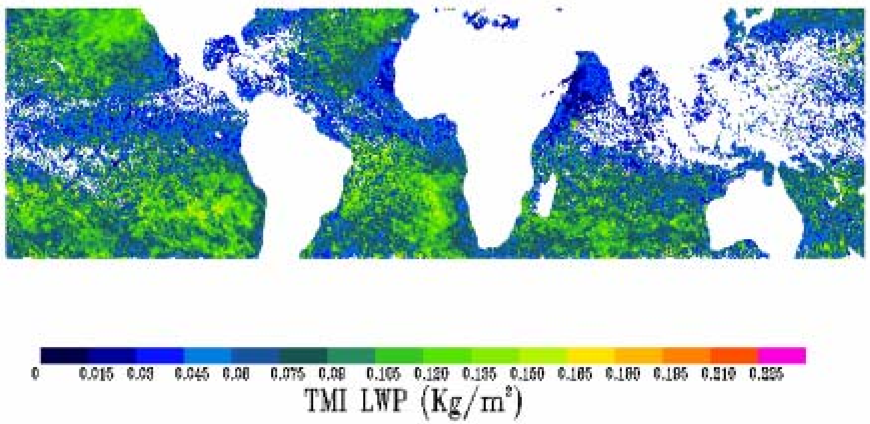


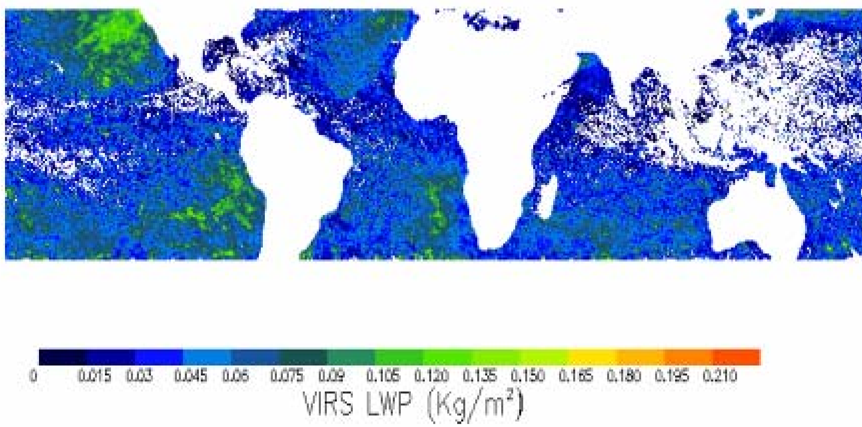
Liquid Water Path (kg/m^2) (July 1990)

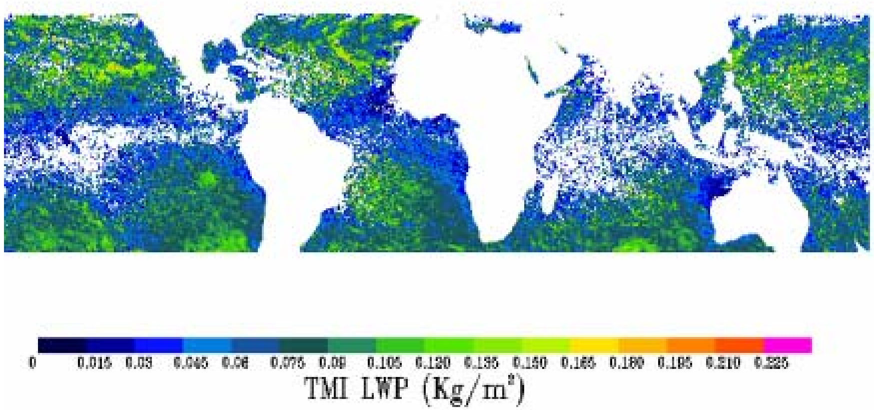


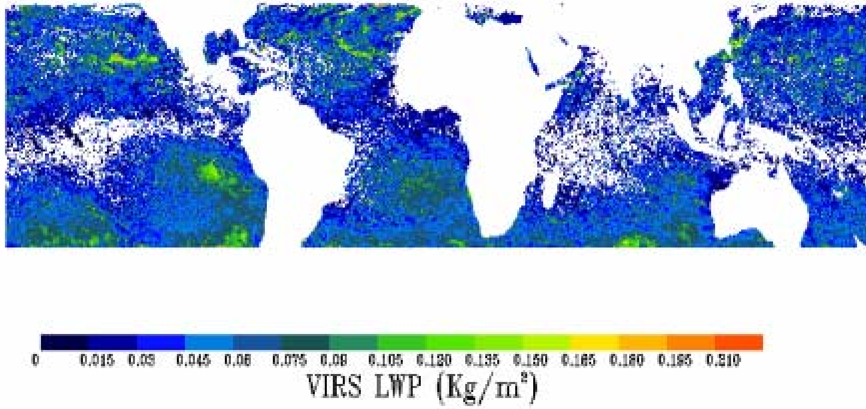
TMI cloud LWP

VIRS cloud LWP

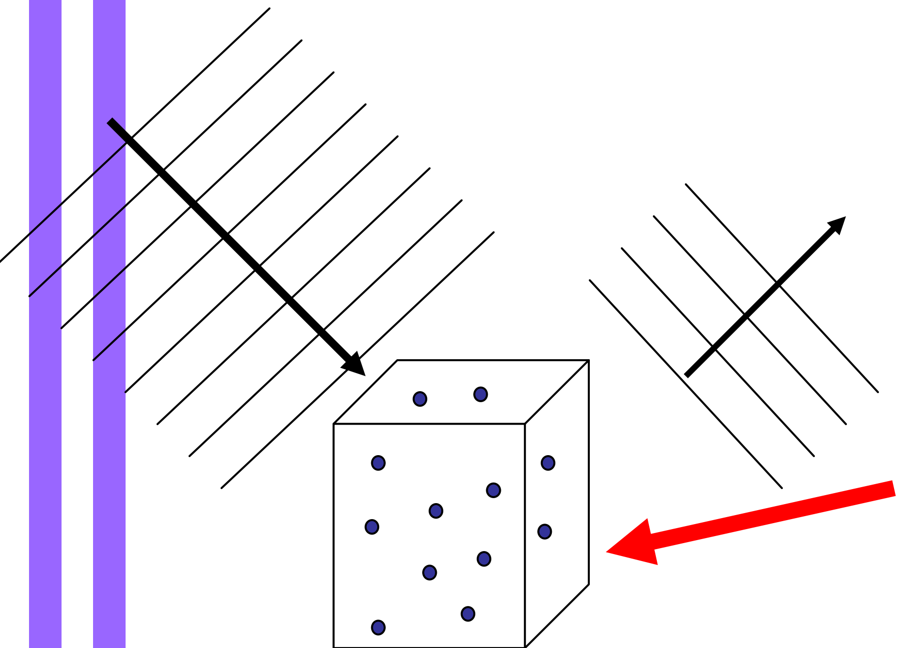








Active systems: the mm radar (e.g. CloudSat)



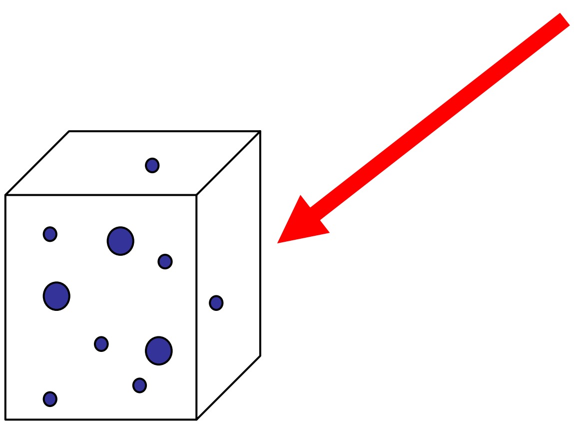
Power returned to radar after being scattered from cloud volume is related directly to size of particles in the volume

For a hypothetical cloud (particles all the same size), the power returned

$$Z = \int n(D) D^6 dD \rightarrow N_0 D^6 \rightarrow \left(N_0 D^3\right)^2$$

is proportional to the square of the water and ice content of the (radar) volume

BUT



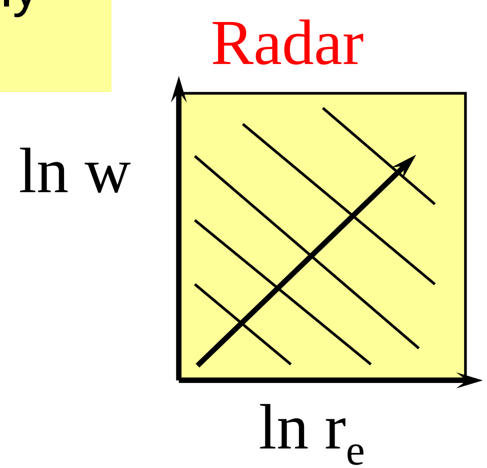
For real cloud (particles in the volume range in size), the power returned (or Z) is *approximately* proportional to the square of the water and ice content of the (radar) volume.

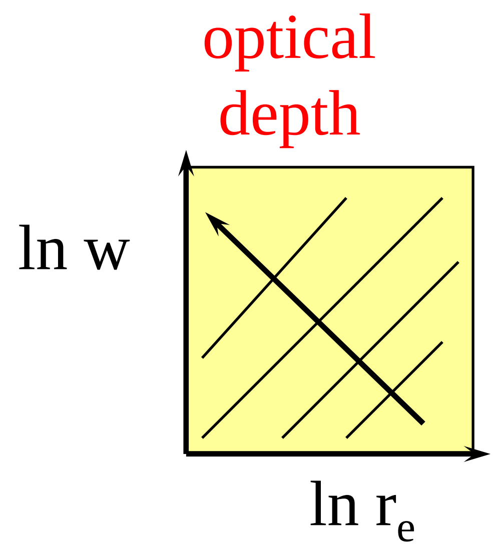
(The CloudSat) Liquid Water content example: the general idea

Active $$Z \rightarrow N_o\, r^6 \rightarrow w\, r_e^3$$

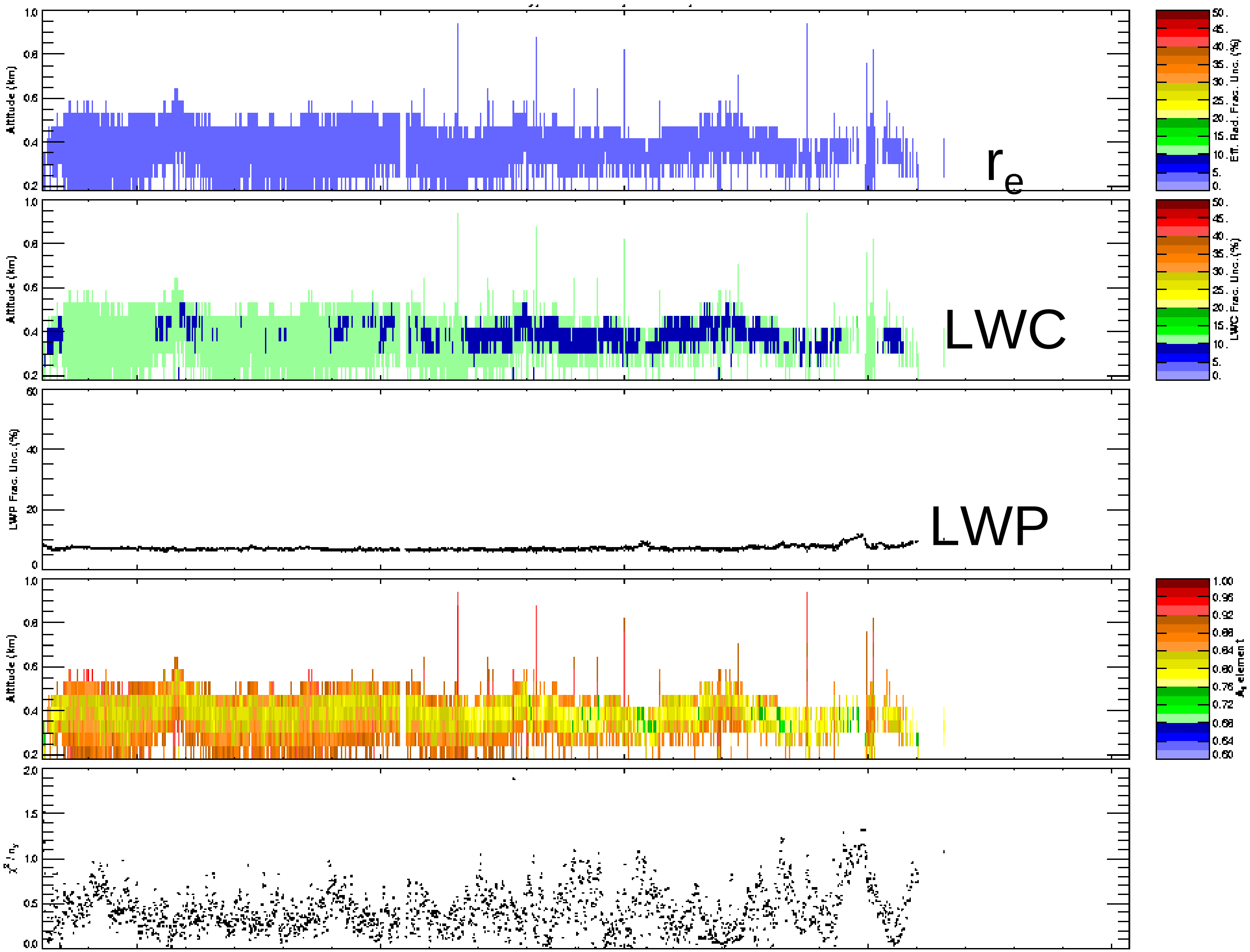
Passive $$\tau \rightarrow \int N_o\, r^2\, dz \rightarrow \int w/r_e\, dz$$

The w-r_e dependency of lidar/τ and radar back-scatter are functionally orthogonal.





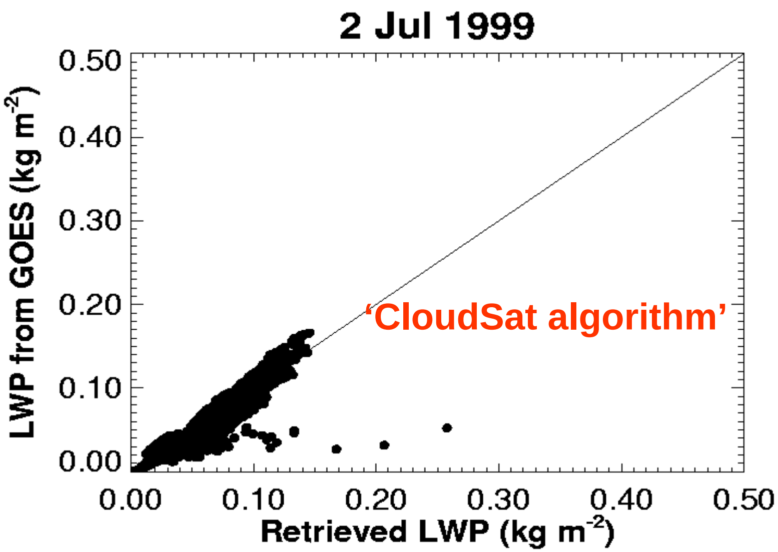
Derived quantities Fractional Uncertainties



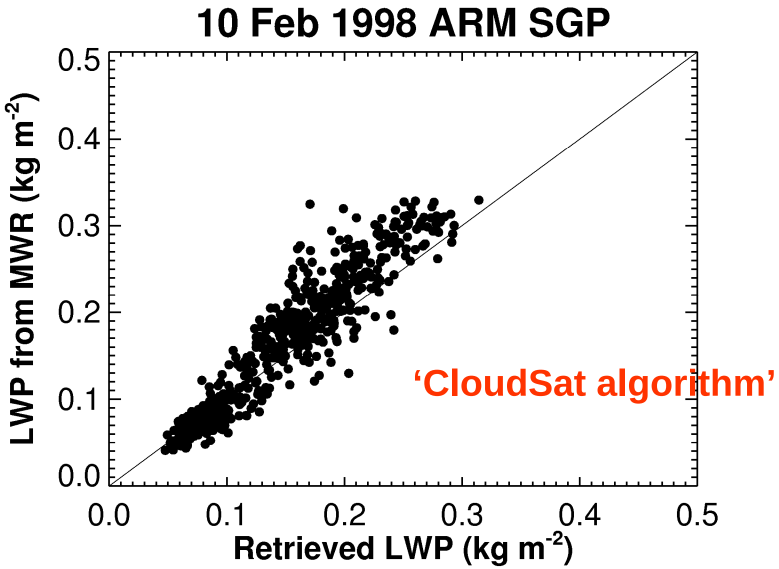
Austin and Stephens, 2001;Austin et al., 2005

Cloud Liquid Water Path

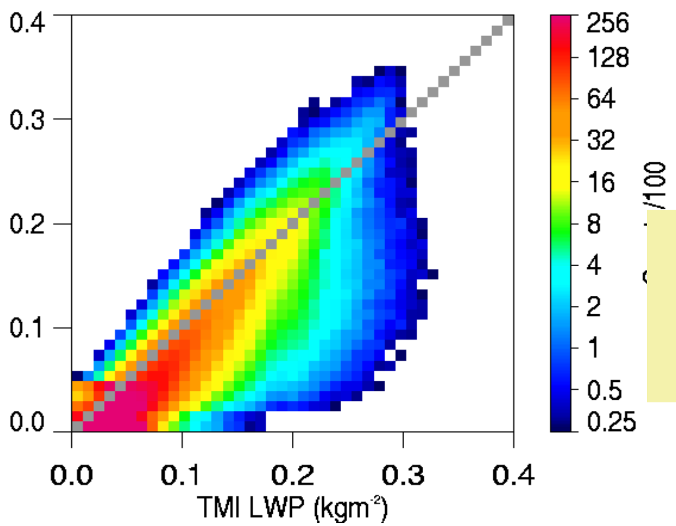
'optical'



microwave



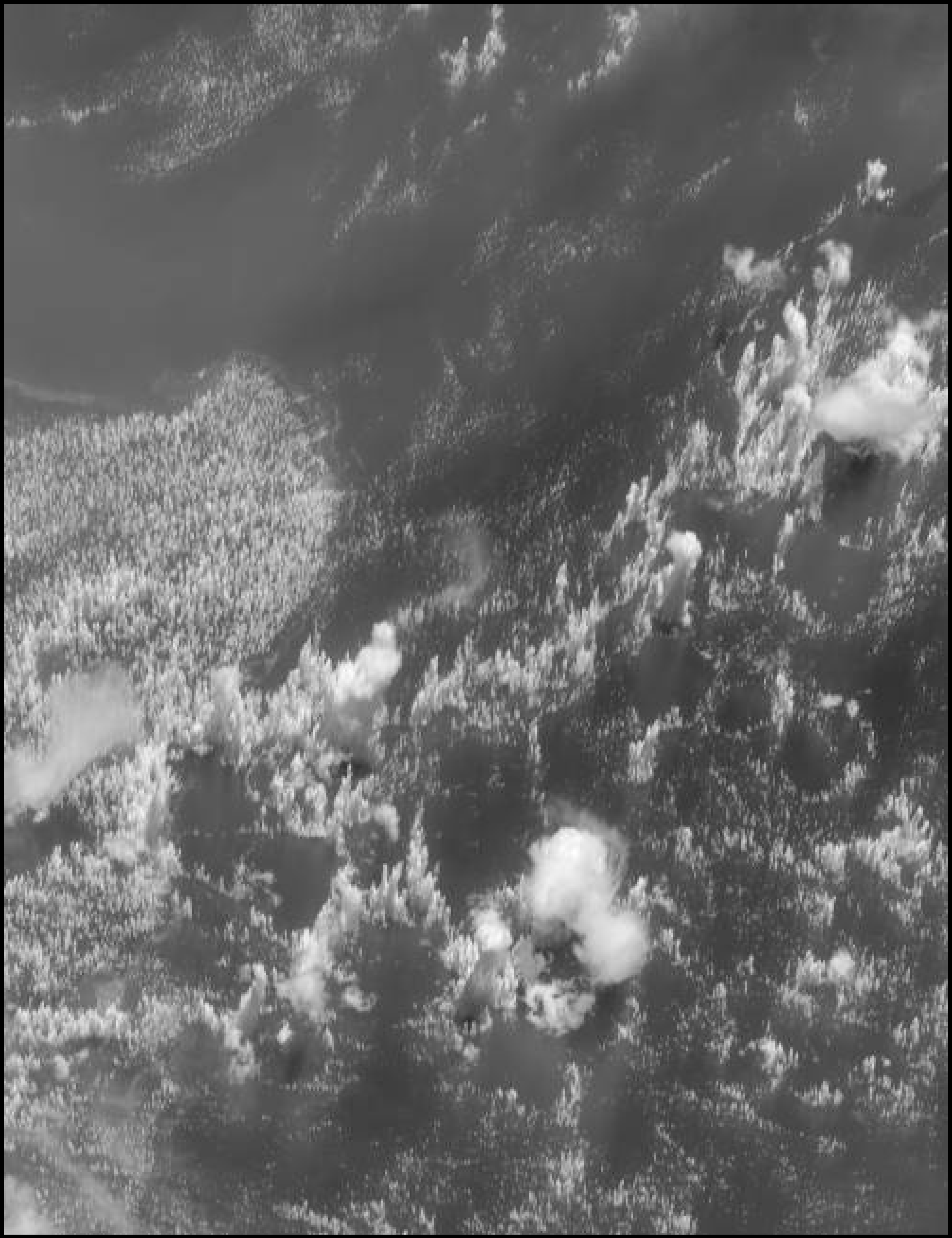
'optical'



microwave

There is a significant bias in the TMI LWP information (→30%) compared to VIRS

Courtesy Greenwald and Christopher



The next dimension - adding vertical resolution

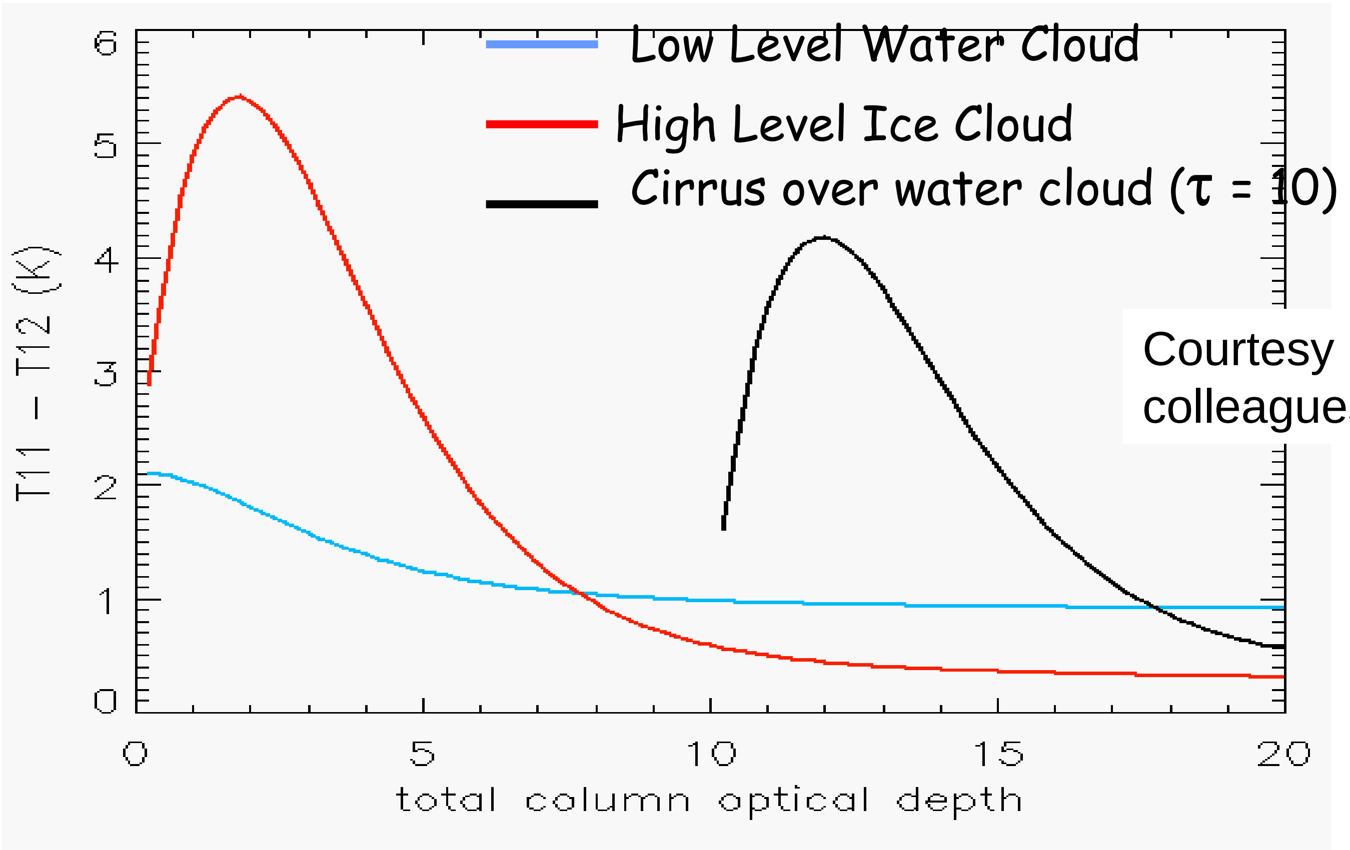
Stereo example from

MISR

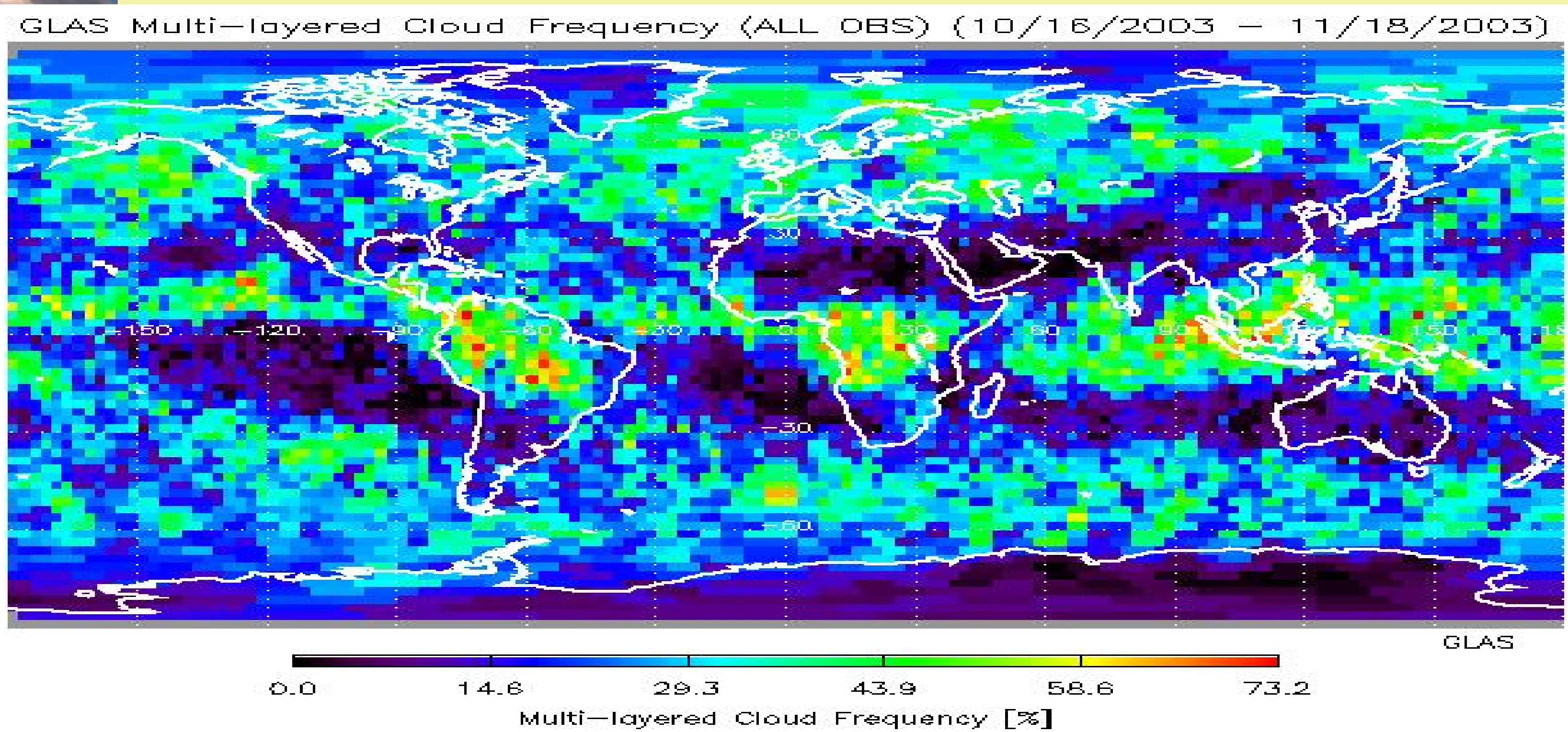
AVHRR Multi-layer Cloud Detection Approach

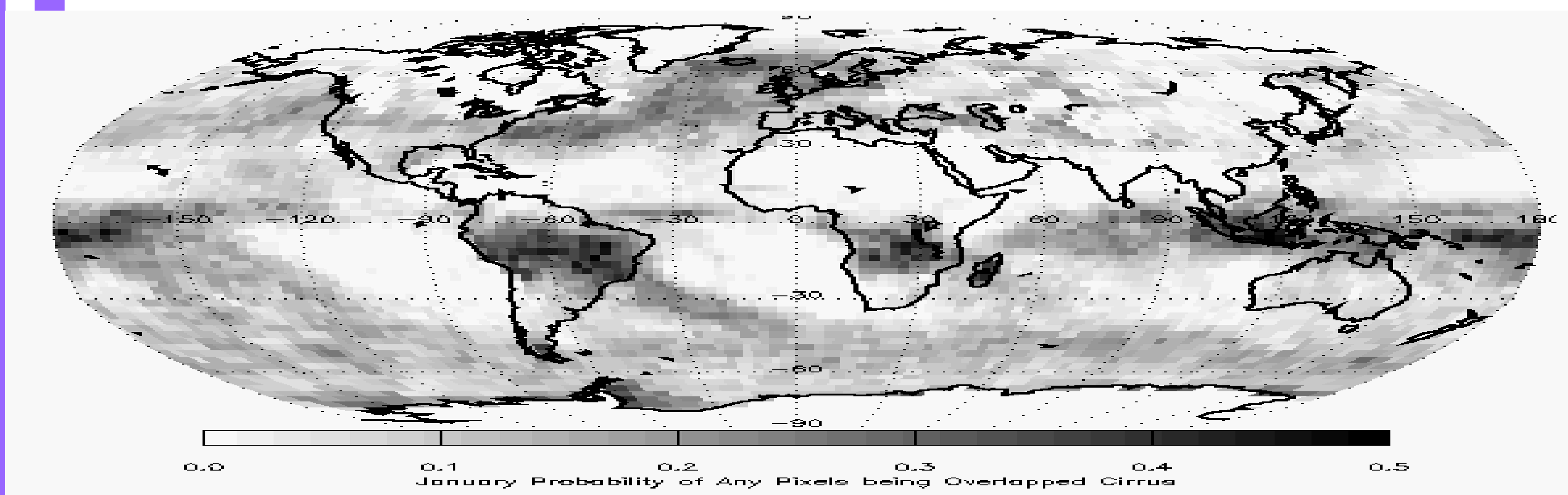
- For single layer clouds, radiative transfer simulation show that as optical depth increase beyond 2, the 11 – 12 micron brightness temperature decreases and approaches an asymptotic value

- Multi-layer clouds exhibit a relationship that can not be modeled (or confused) assuming single layer clouds.



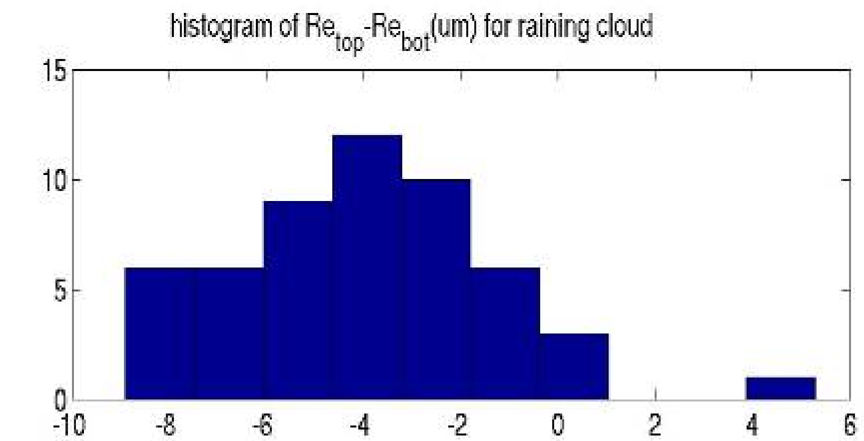
Courtesy Heidinger & colleagues

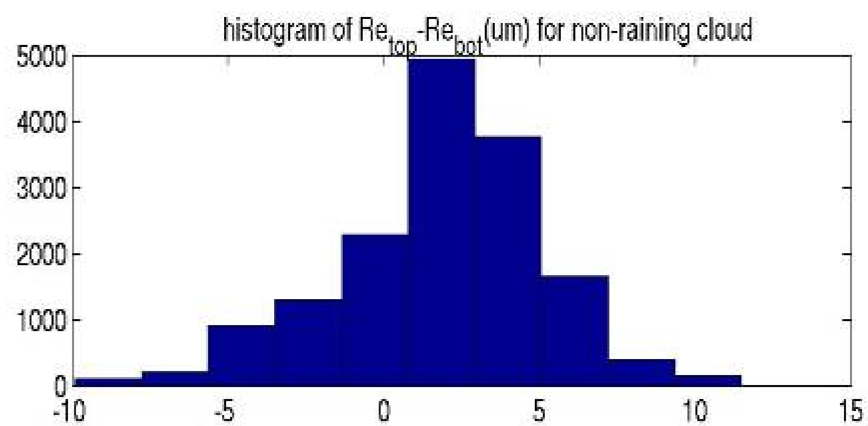

GLAS Multi-layered Cloud Frequency (ALL OBS) (10/16/2003 – 11/18/2003)
GLAS
0.0 14.6 29.3 43.9 58.6 73.2
Multi-layered Cloud Frequency [%]


0.0 0.1 0.2 0.3 0.4 0.5
January Probability of Any Pixels being Overlapped Cirrus

Particle size 'profile' retrieval

- Raining Cloud, mostly, Re_top<Re_base
- Re_top>Re_base could happen for raining cloud because of the non-raining part within the pixel
- For non-raining cloud, most R_top>R_bot
- R_top<R_bot could happen for raining cloud because th cloud particle is too small tc form rain or the rainfall is to weak for microwave detecti



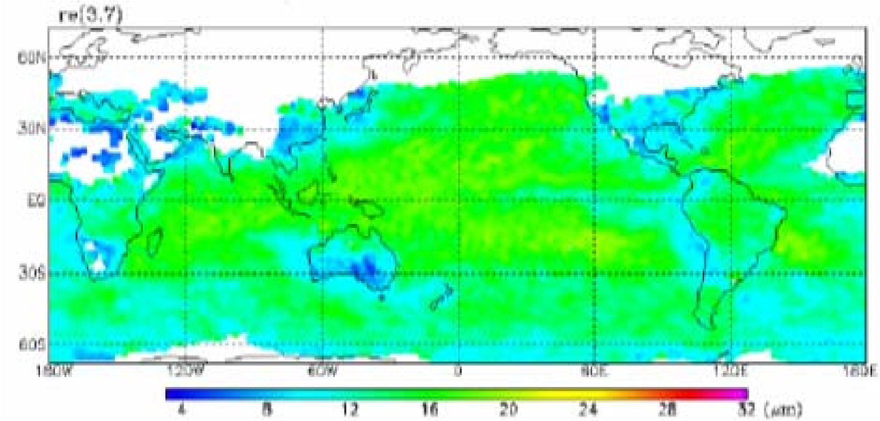


Chang & Li, 2002;2003

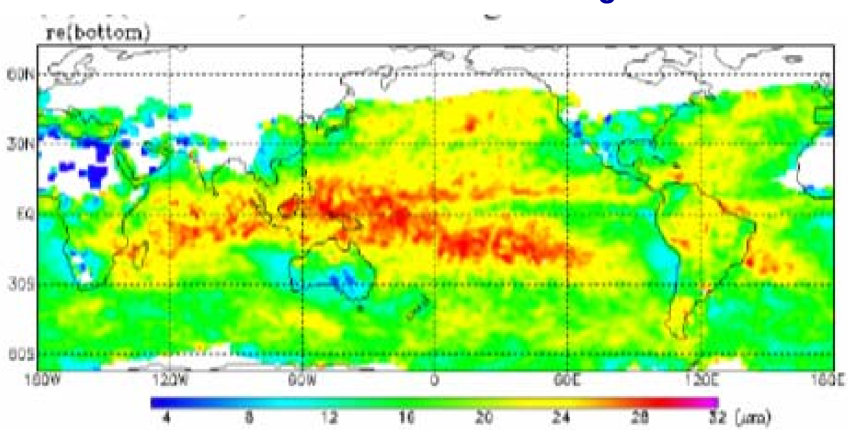
MODIS Retrieved Cloud-top and Cloud-bottom r_e and TRMM Rainfall Data

Cloud-top r_e

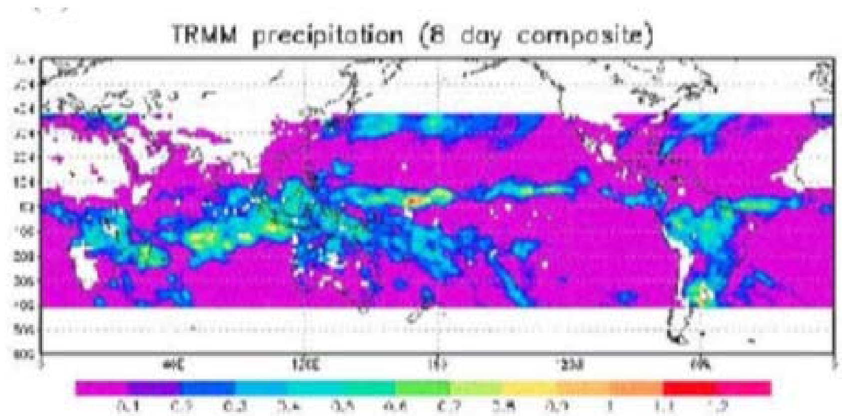
Chang & Li, 2002,2003



Cloud-bottom r_e

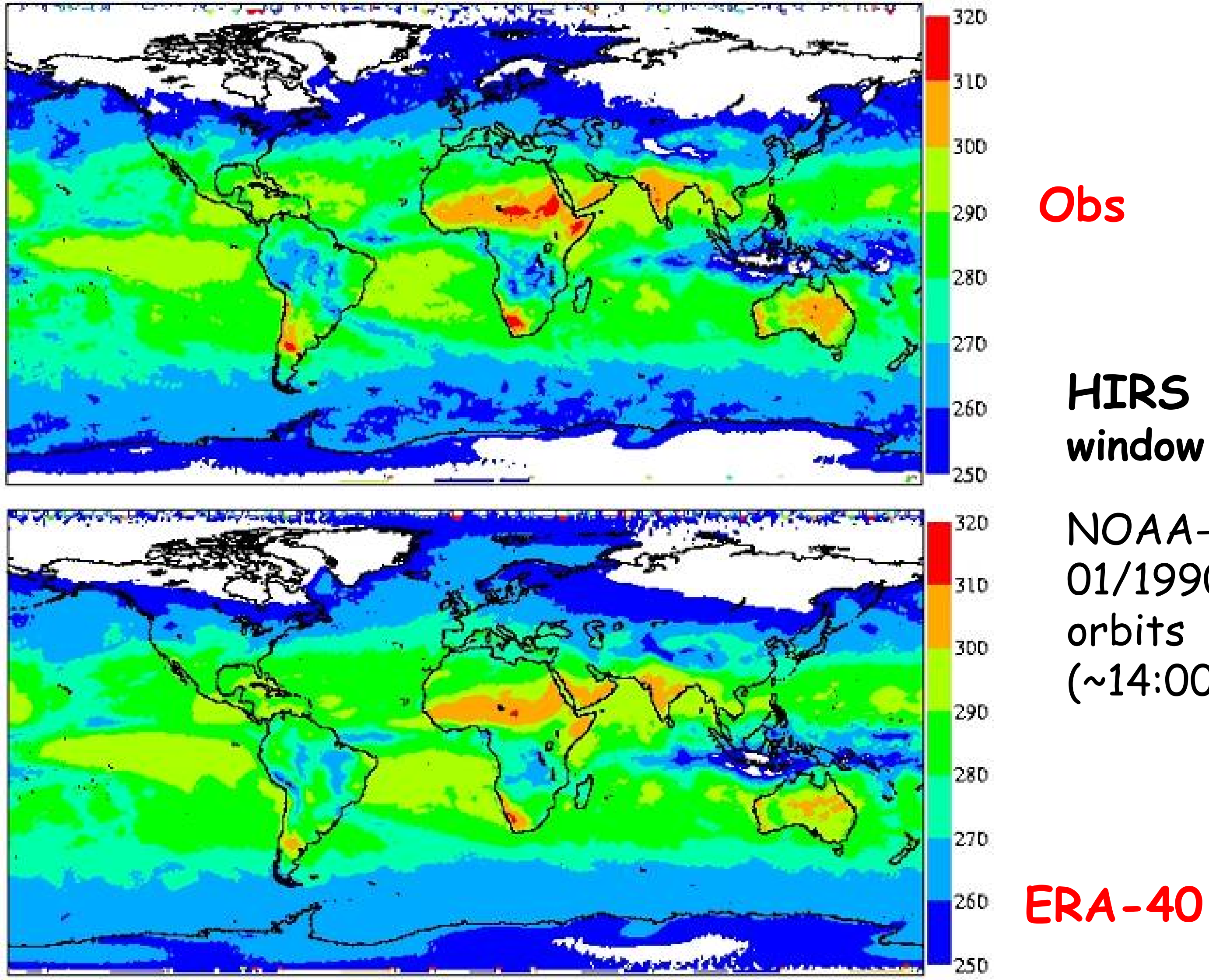


TRMM rainfall rate



Model evaluation

Model vs. HIRS 11 μm window (K)

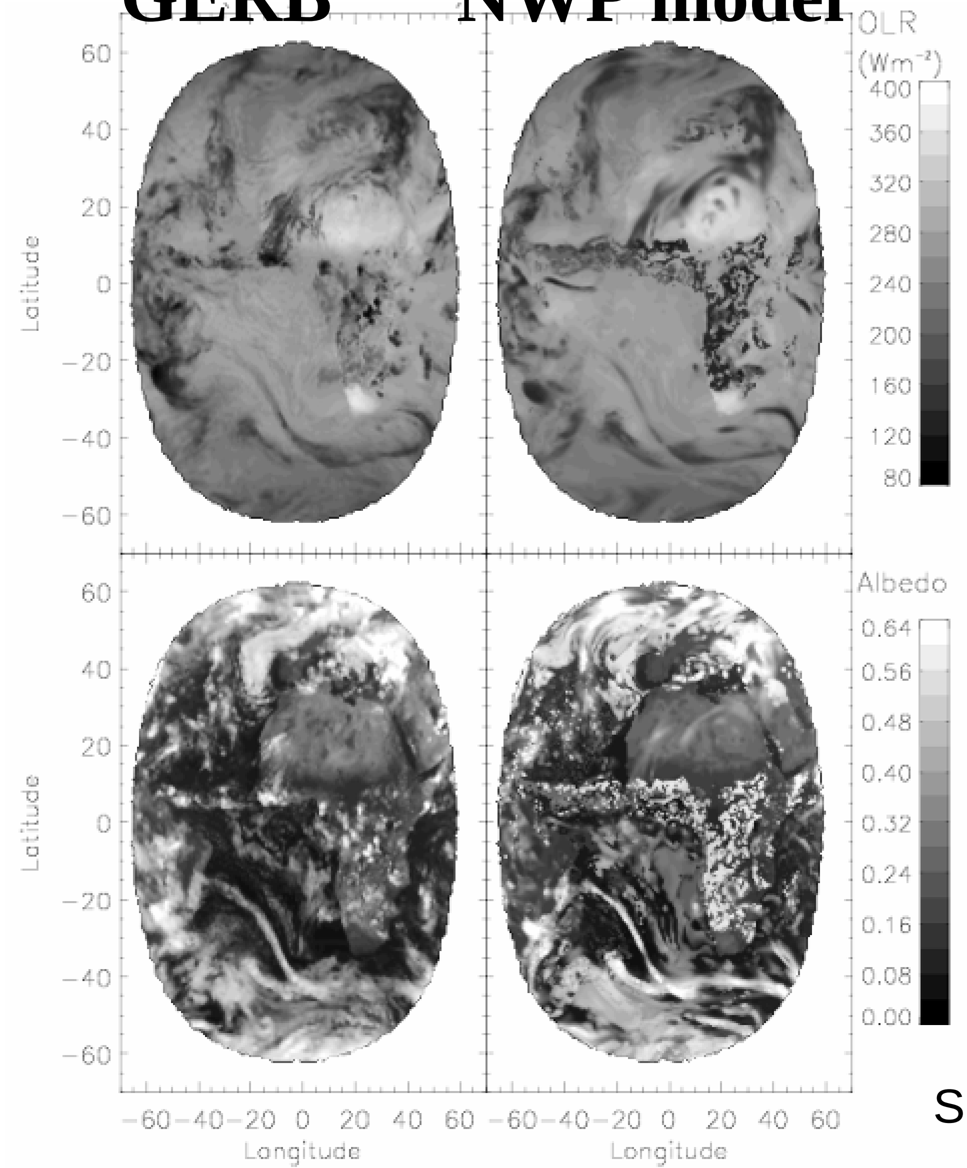


HIRS 11μm window (K)

NOAA-11
01/1990 PM orbits
(~14:00 LT)

Model-observation comparison

GERB **NWP model**



O L R

A L B E D O

Recent comparison

15 November 2004

1200 UT

Model cloud errors can easily be distinguished. Near-real time comparisons are valuable for a wide range of other studies (e.g. outbreaks of Saharan dust)

Slingo et al, 2005

ISCCP histogram-cluster analysis (Jakob and Tseloudis)

TWP

Manus ECMWF

Frequ.= 33 %

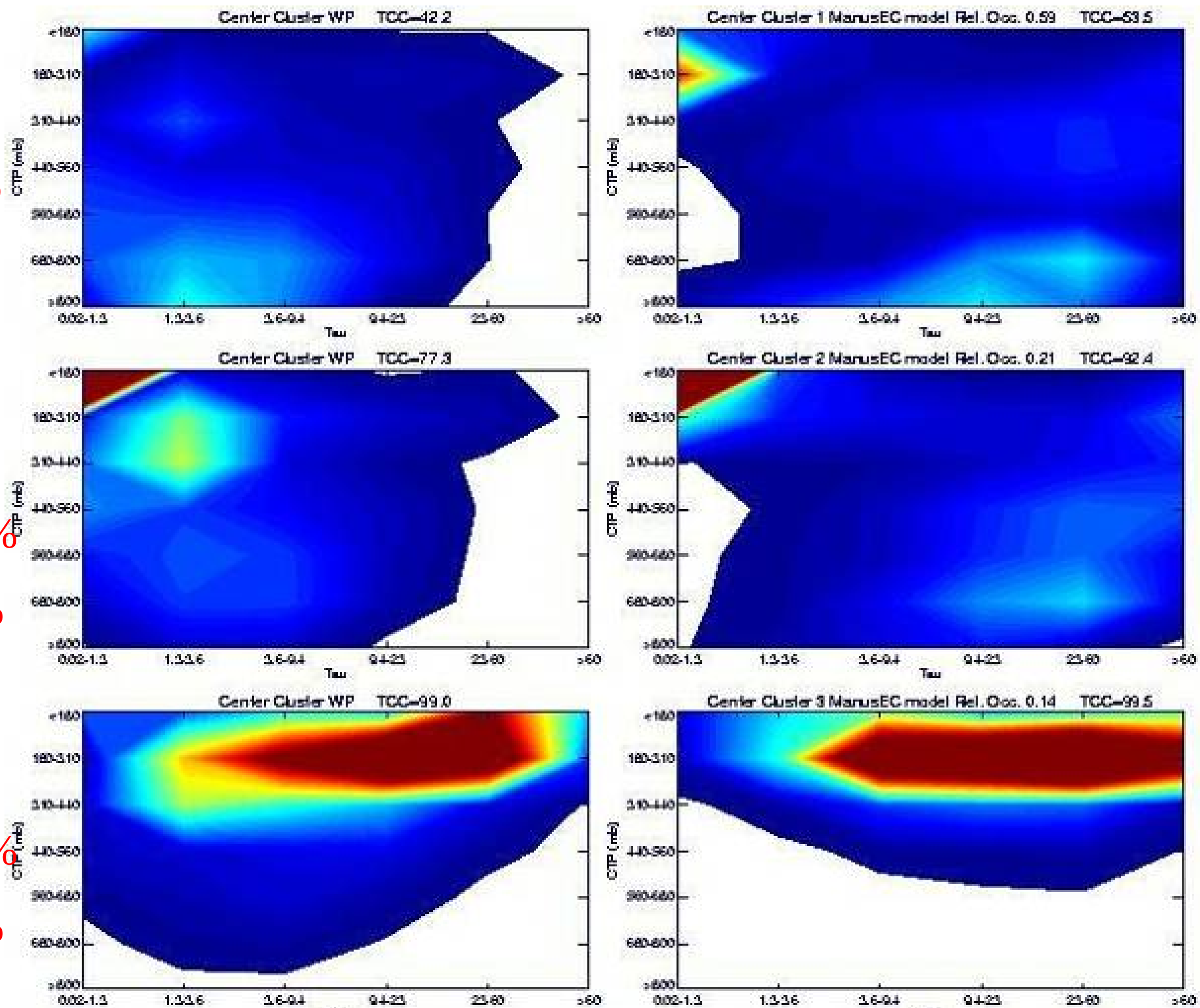
TCC= 32 %

Frequ.= 33 %

TCC= 75 %

Frequ.= 11 %

TCC= 99 %



Frequ.= 59 %

TCC= 53 %

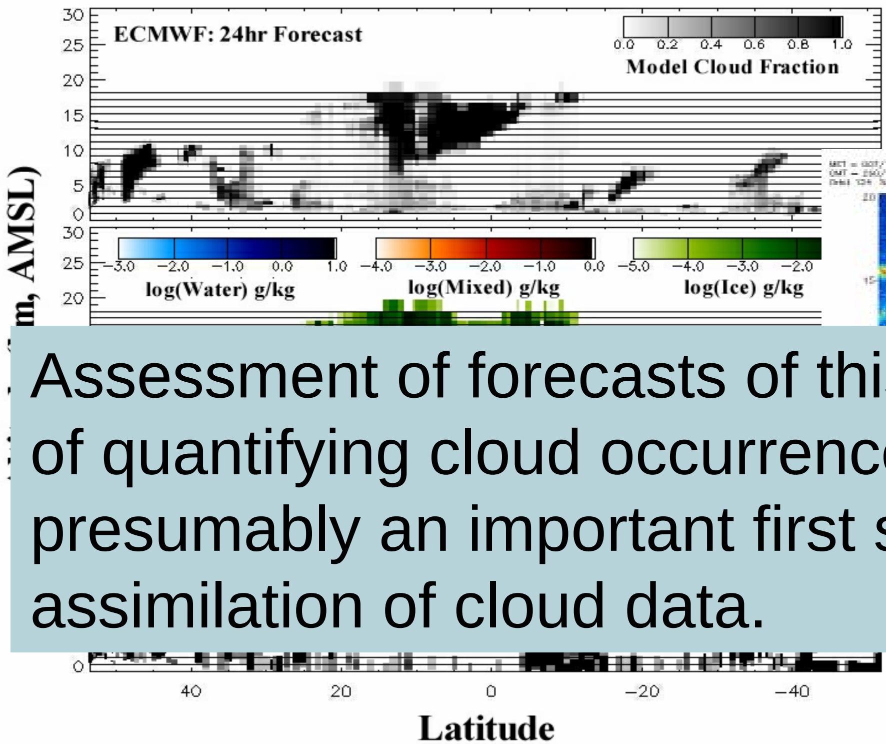
Frequ.= 21 %

TCC= 92 %

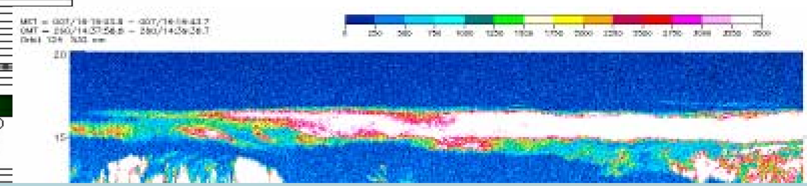
Frequ.= 14 %

TCC= 99 %

Example of the use of orbit data for evaluating NWP model predicted cloudiness



Miller et al., 2000,GRL



Assessment of forecasts of this nature, even just in terms of quantifying cloud occurrence model errors, is presumably an important first step toward eventual assimilation of cloud data.

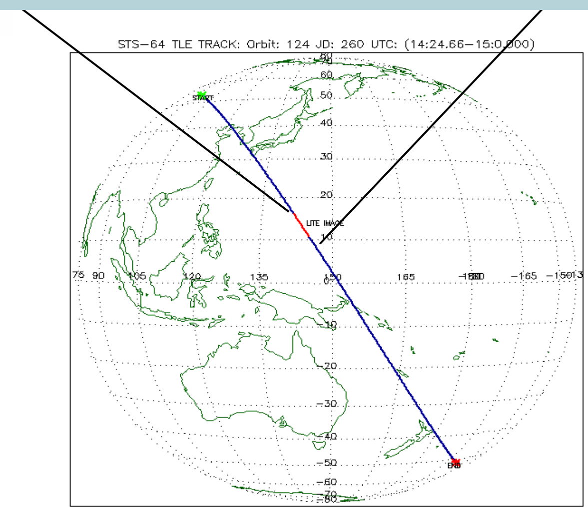
ECMWF/LITE correlative study Statistics for 60+ LITE Orbits, ±1 bin horizontal and vertical

Hit Rate = fraction cloudy+clear correctly forecast, =0.896

Threat Score = fraction of cloud points correctly forecast = 0.714

Probability of Detection = ratio of cloud hits to total # of obs clouds = 0.796

False Alarm rate = rate of forecasting cloud when clear = 0.126



Summary

Many satellite measurements offer redundant information about clouds and precipitation. This is good for the purpose of cross-comparing information as a step to validating knowledge but we cannot be confident about knowing if we are approaching a truth and we have not articulated a clear path to do so.

There is generally little rigor in uncertainty analysis attached to cloud products (if it exists at all), mostly because uncertainties are difficult to validate. This leads to many problems:

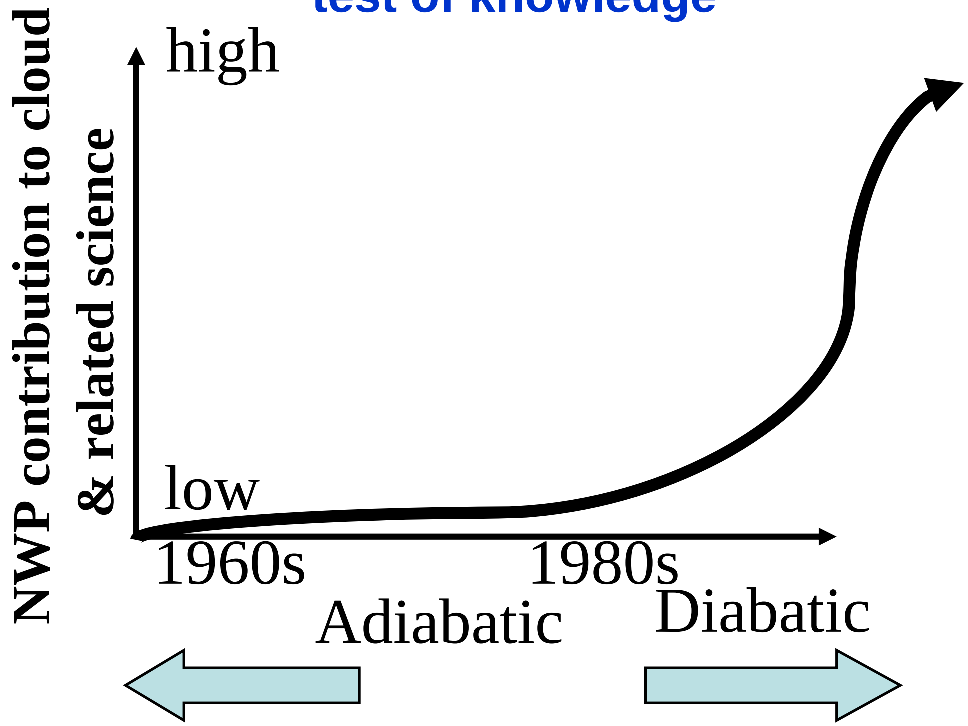
- We cannot make meaningful judgments about which of the different approaches is most accurate,
- We have little basis for arguing for small changes in key parameters as being real (e.g. cloud trends)
- We cannot determine the value of combining different measurements such as from multi-sensor observing systems,
- We cannot meaningfully assimilate the observations into dynamical systems

As we enter an era of the grand challenge, an era of multi-sensor integration and data assimilation, it becomes essential that we develop tools that:

1. Determine more precisely what information resides in measurements of different types as a step to better use of them,
2. Optimally mix information from multiple sources of measurements, and
3. Convert this optimal information to knowledge through (at a minimum) quantification and validation of errors

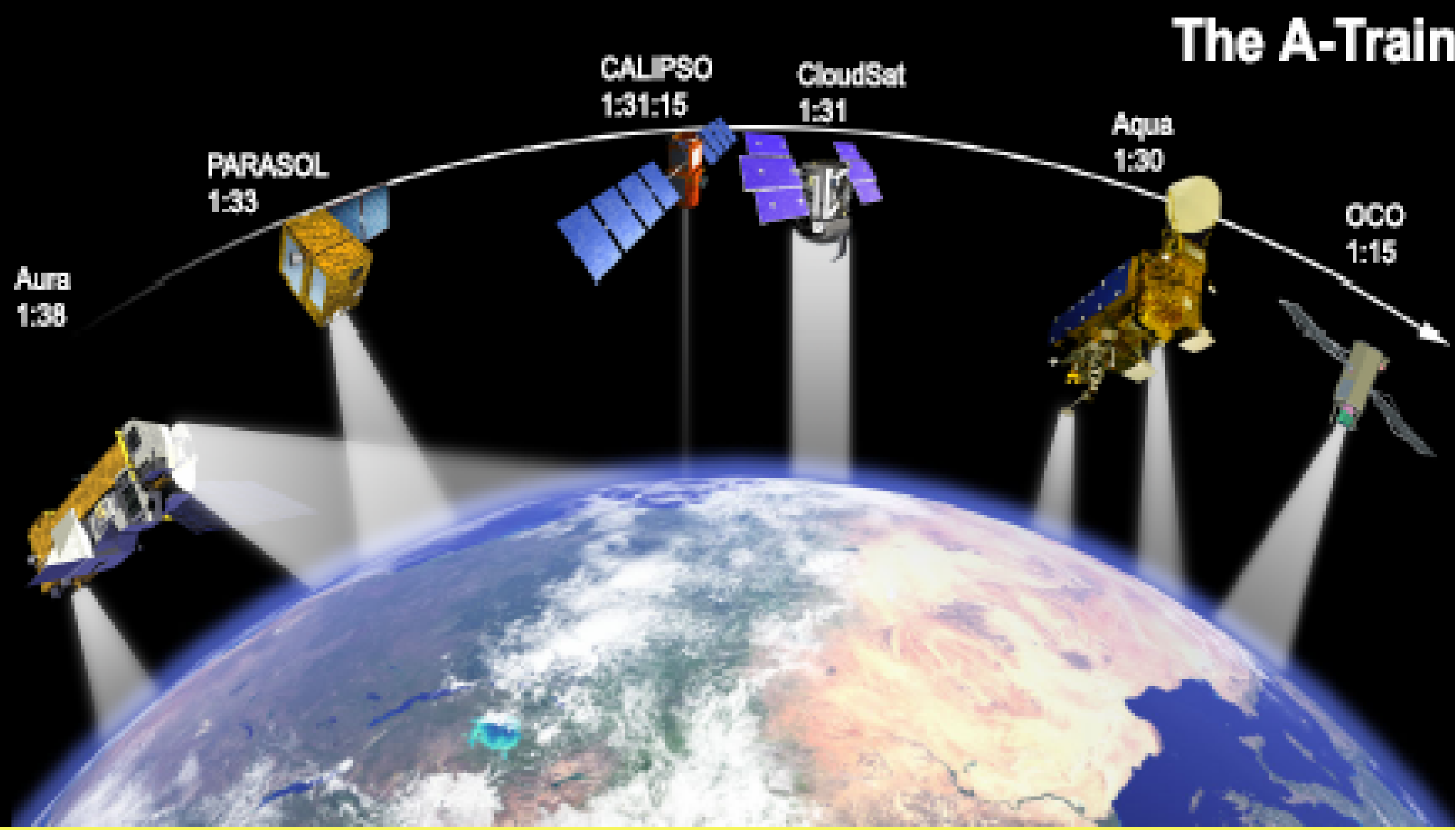
This is a period of great optimism but much is left to be done.

Better NWP cloud predictions → richer assimilation →more rigorous test of knowledge



…. Well, I think one could always devote more effort. Effort by itself isn't enough, I think inspiration is also important!

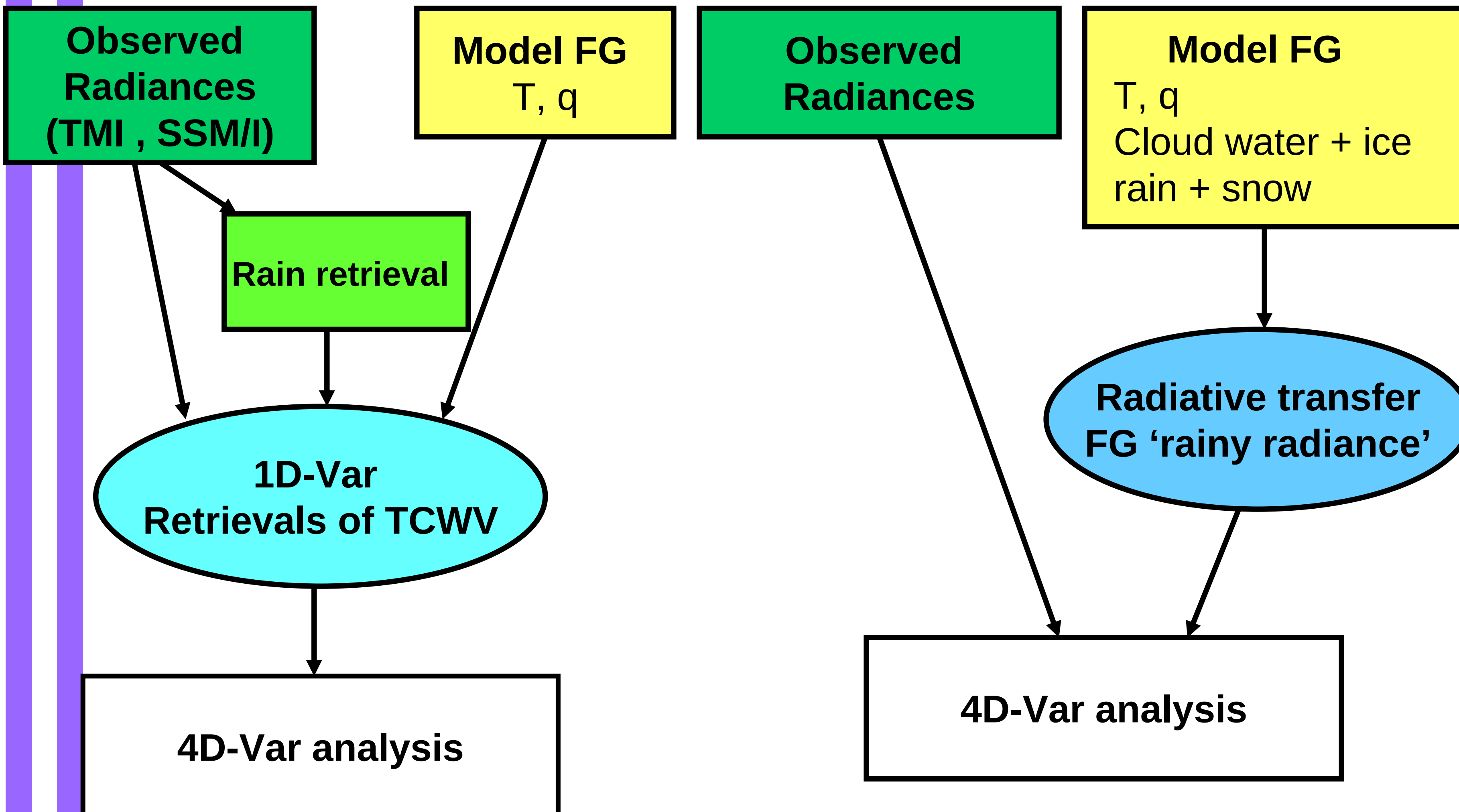
Charney to Platzmann



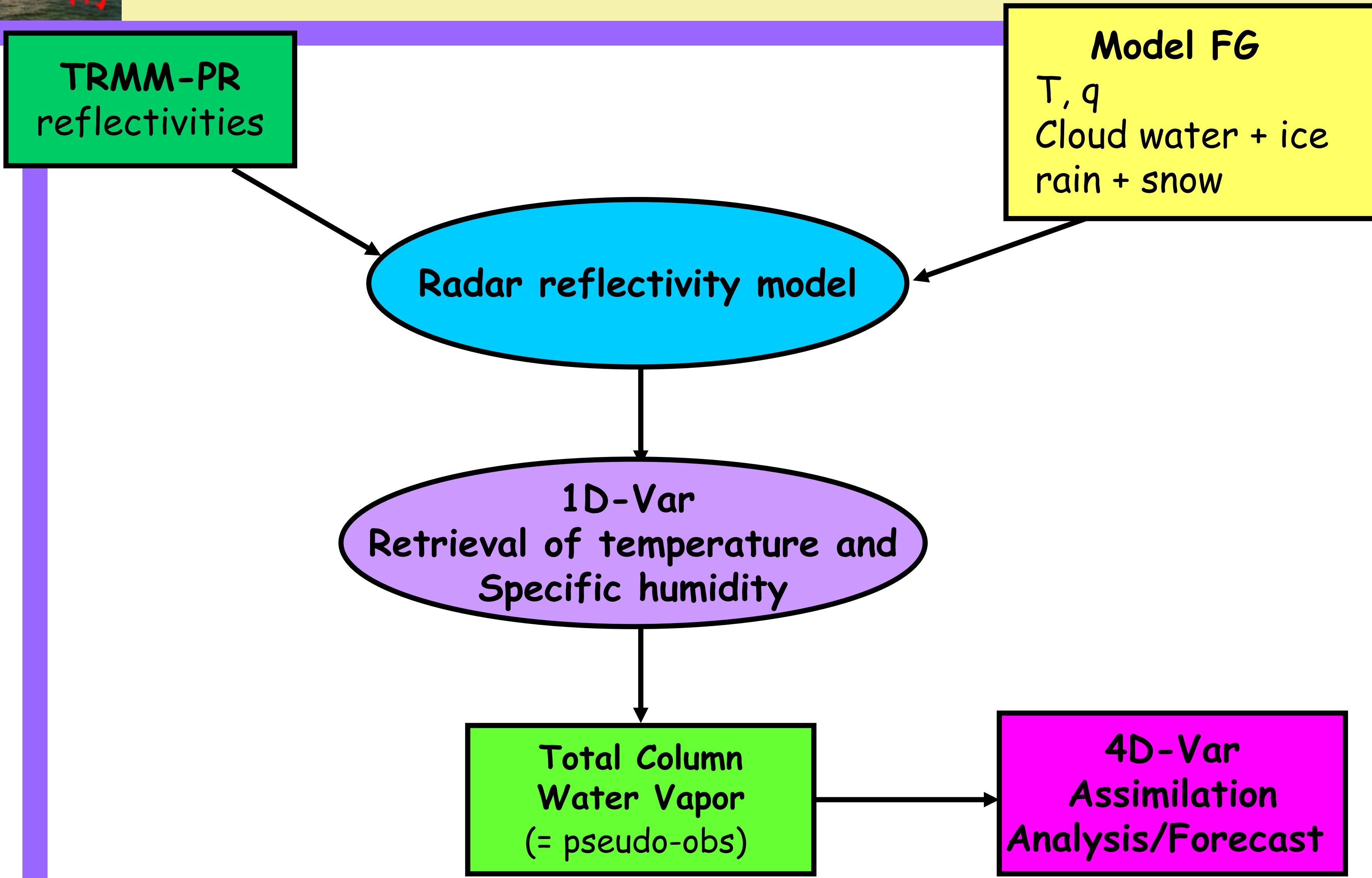
By mid 2005, we expect to have a wide range of different sensors, active and passive, optical, infrared and microwave, hyper-spectral to coarse band, all approximately viewing Earth at the same time. We are left to pose a strategy that optimally combines these measurements, converting them to meaningful information with verified uncertainties.

Backup

Alternative approaches for assimilation of rain information

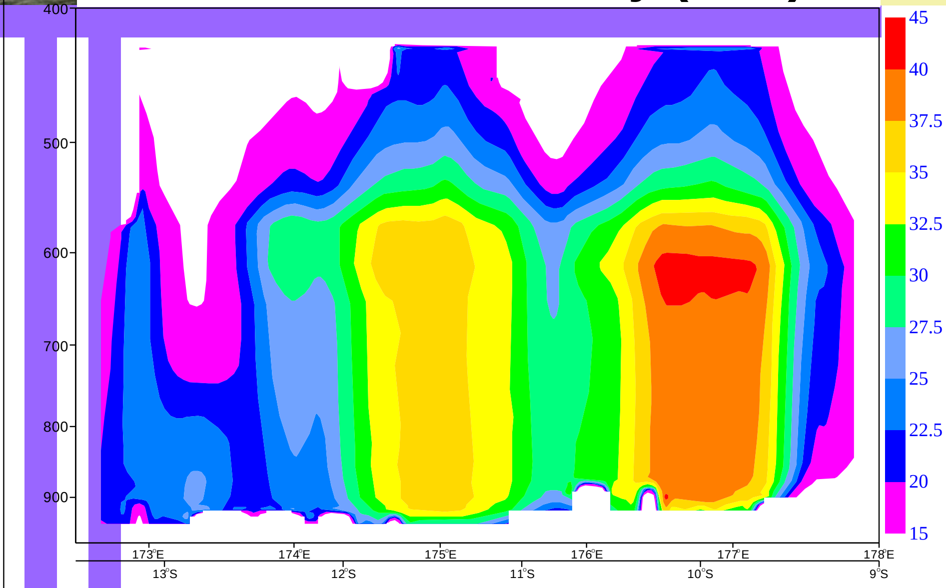


1D+4D-Var on TRMM/PR reflectivities



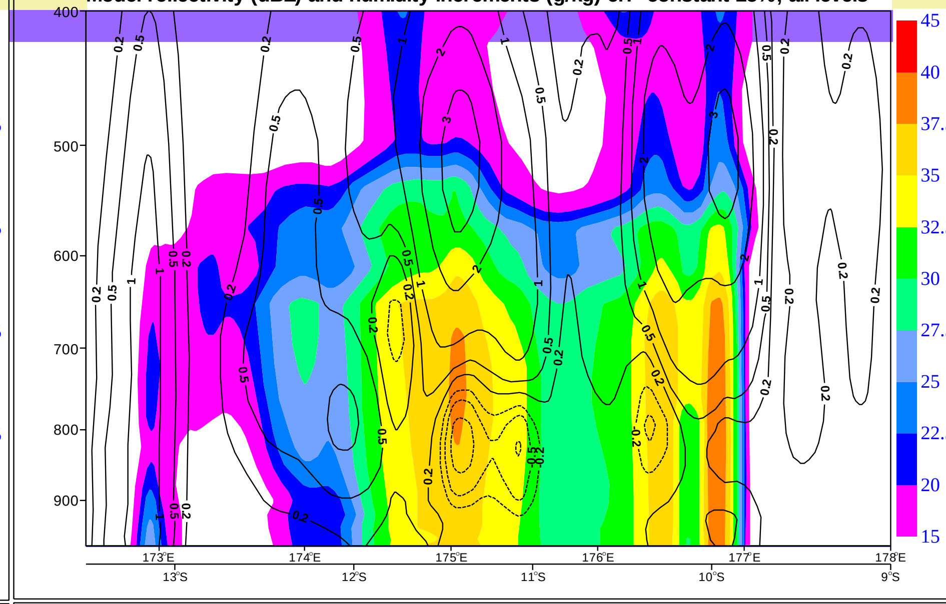
1D-Var retrievals using PR reflectivities with different error assumptions on PR-Z

TRMM PR reflectivity (dBZ)

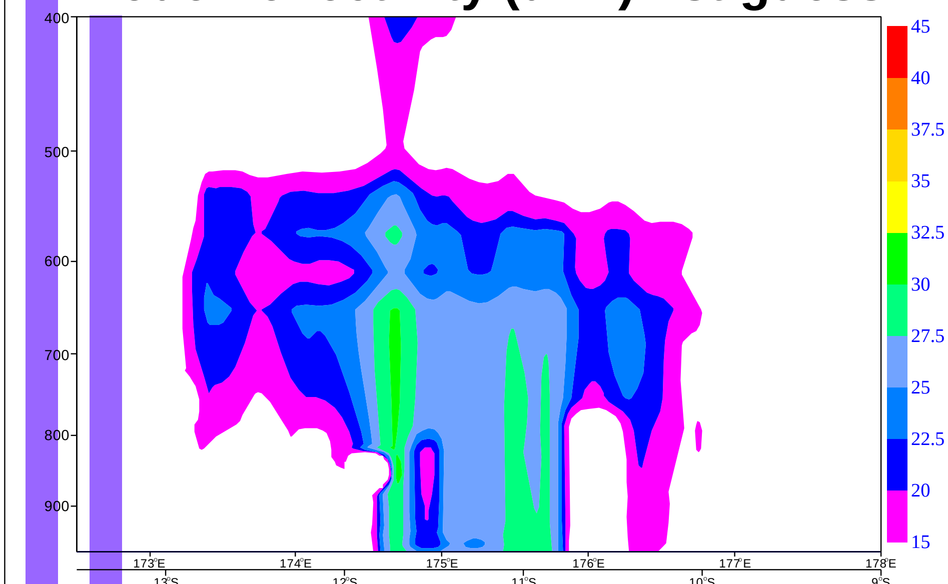


1D-Var 25% error at all levels

Model reflectivity (dBZ) and humidity increments (g/kg) err=constant 25%, all levels

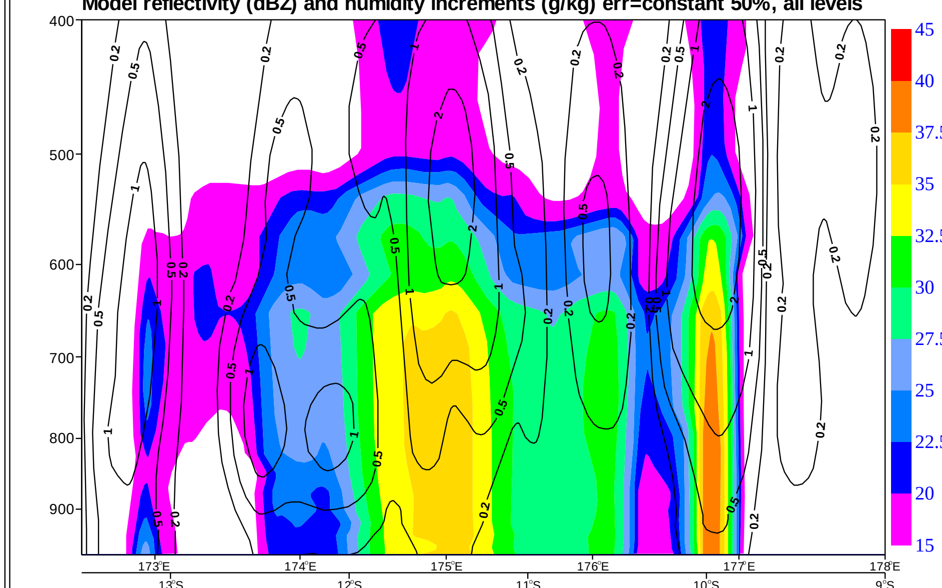


Model reflectivity (dBZ) fist guess



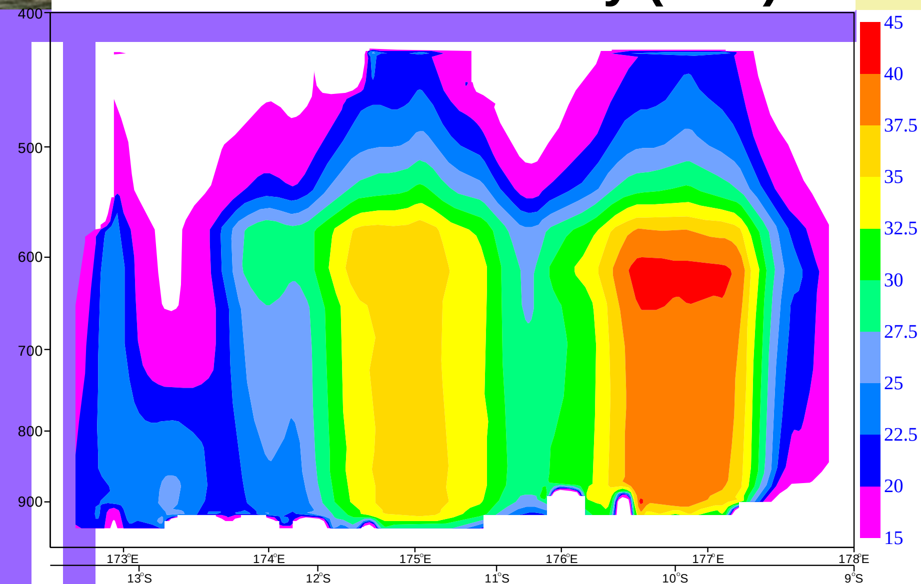
1D-Var 50% error at all levels

Model reflectivity (dBZ) and humidity increments (g/kg) err=constant 50%, all levels



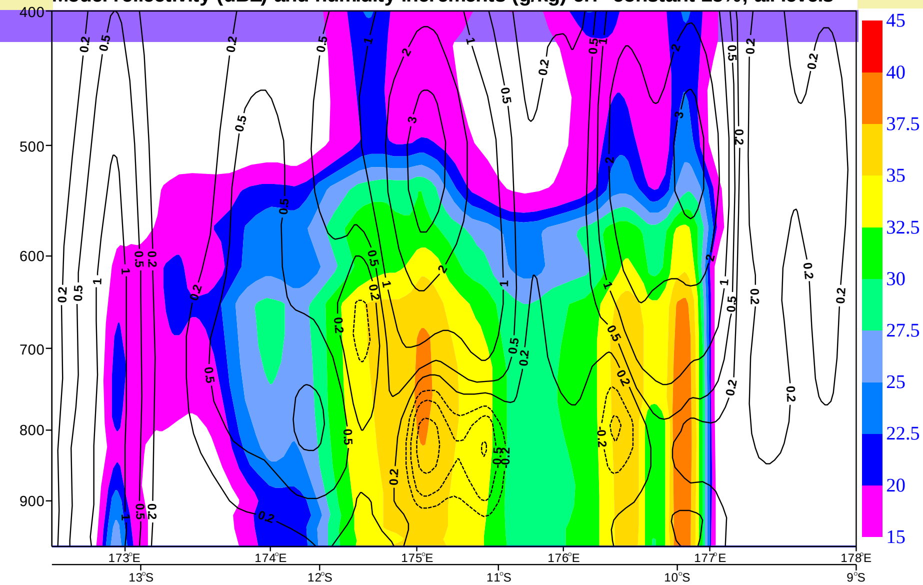
1D-Var retrievals using PR reflectivities: observations at one level only vs full profile

TRMM PR reflectivity (dBZ)

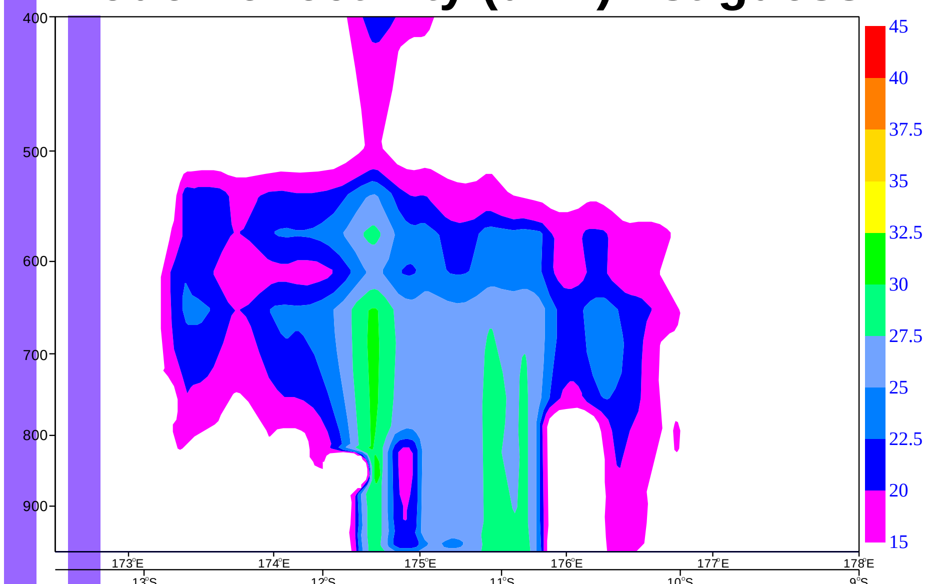


1D-Var obs at all levels

Model reflectivity (dBZ) and humidity increments (g/kg) err=constant 25%, all levels

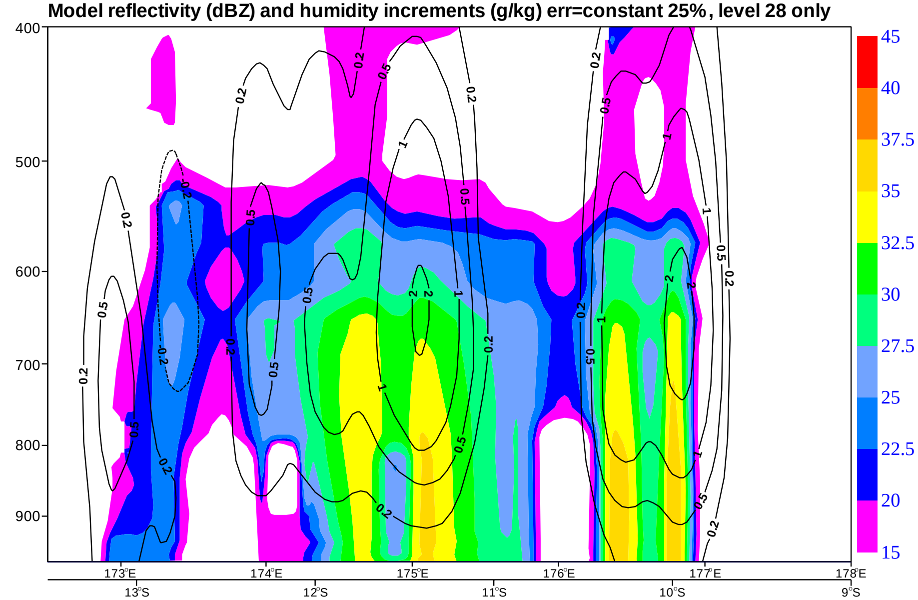


Model reflectivity (dBZ) fist guess

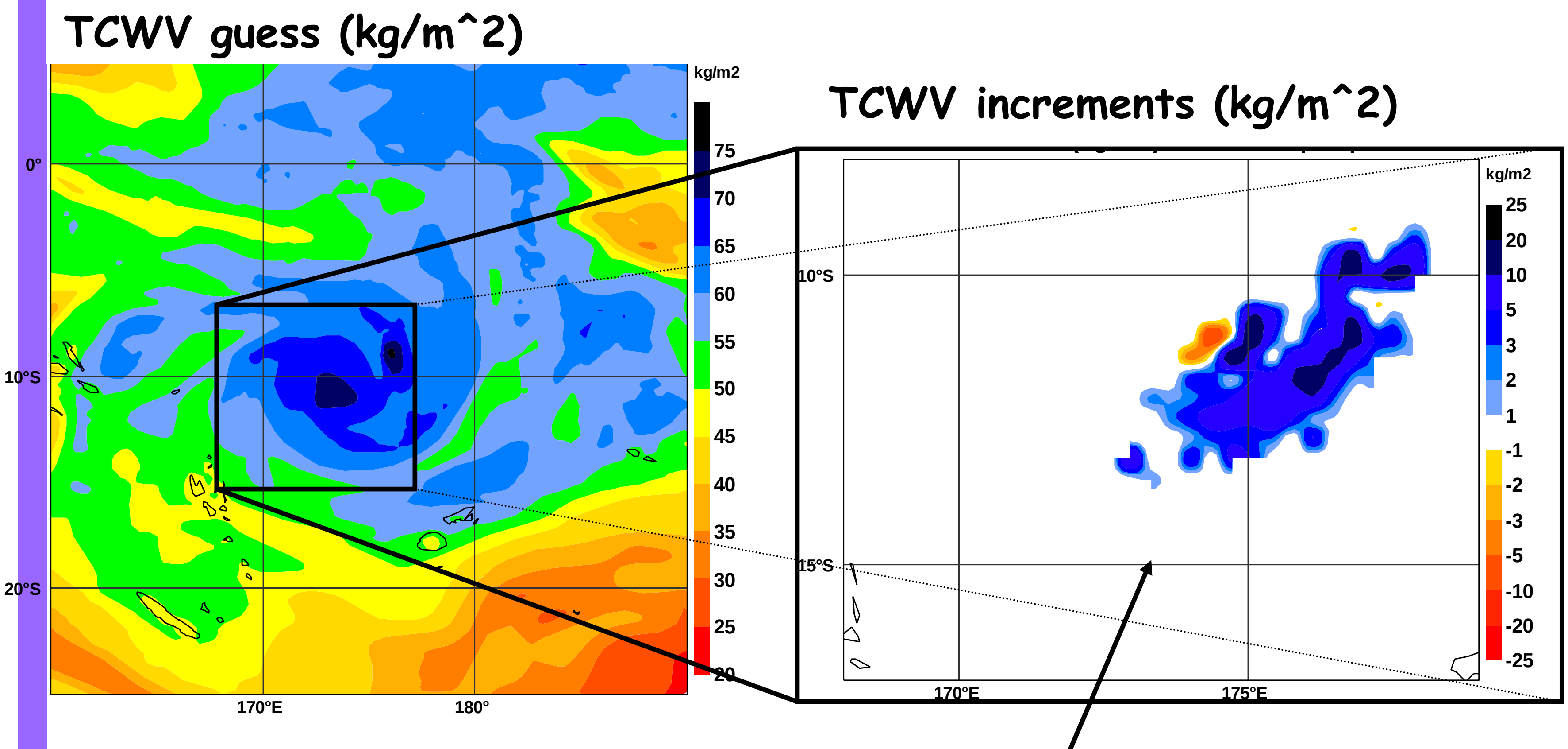


1D-Var obs at level 48 (~2km)

Model reflectivity (dBZ) and humidity increments (g/kg) err=constant 25%, level 28 only



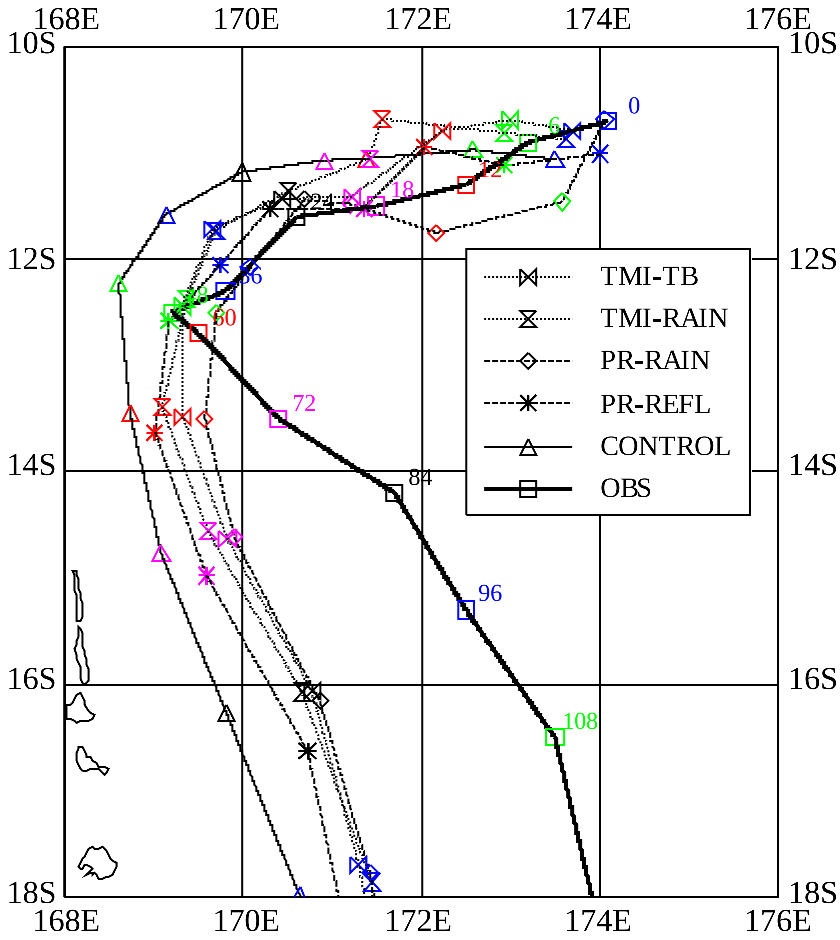
Background and 1D-Var increments of Total Column Water Vapour (pseudo-obs for 4D-Var)



Increments indicate an overall moistening confined along the satellite track

Comparison of track forecasts (started on 26 December 2002 at 1200 UTC) obtained from the control, two TRMM/PR, and two TMI experiments to the observed track.

ZOE TRACK FORECAST (BASE: 2002122612)



- As suggested by the MSLP changes, the track forecasts are substantially improved when TRMM observations are assimilated in rainy areas.
- Despite the smaller spatial coverage of TRMM/PR data (200-km swath) compared to that of TMI data (780-km swath), the impact of both types of observations is comparable.

Concluding comments:

1. The assimilation methods pioneered at the Centre represents an important a bridge linking the traditional factions of the sciences.

2. While assimilation of data on quantities characterized as smooth and continuous, we are now entering a period of assimilation of hydrological parameters

And then, of course, there remains, even in the short-range problem, I think, the physical factors, which are still not adequately understood. The matter of the boundary layer and precipitation process .... Charney to Platzmann

1970

The first phase: the period of great imagination

1980

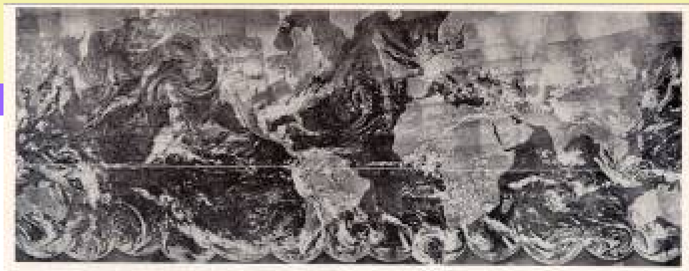
The second phase: the period of great information-gathering

1990

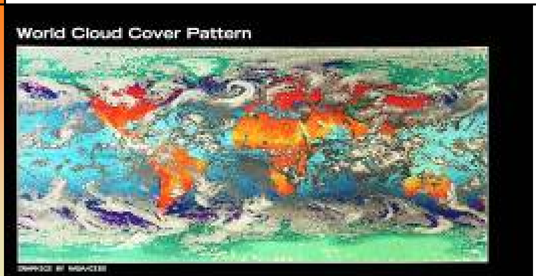
2000

The third phase: grand challenge to create ‘knowledge’

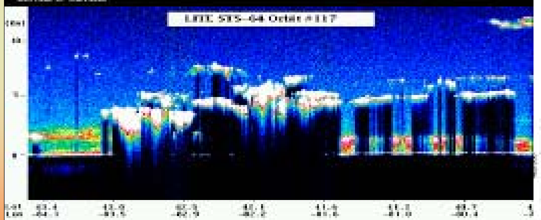
The launch of TIROS-1, April 1960



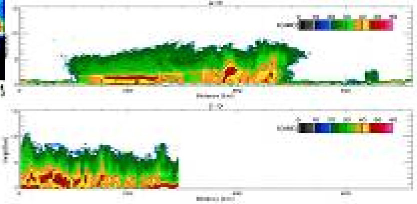
The first 24hr view of global clouds TIROS-9, February 13, 1965



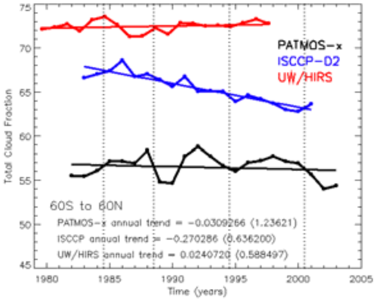
1. Global climatologies of cloud occurrence*, optical properties, 1983-present * Cloud mask/ identification /screening



First flight of back-scatter lidar, LITE, 1996



First flight of precipitation radar, TRMM, 1997



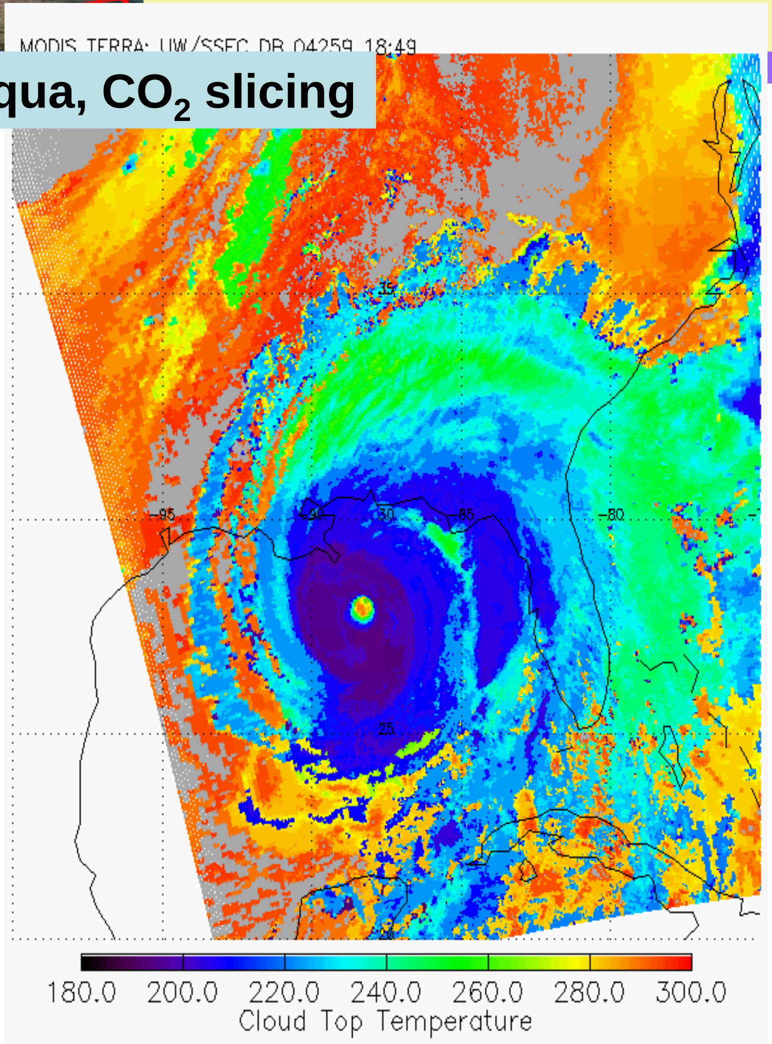
Decadal cloud amount trends, precipitation variability

Assimilation of precipitation and cloud radiances

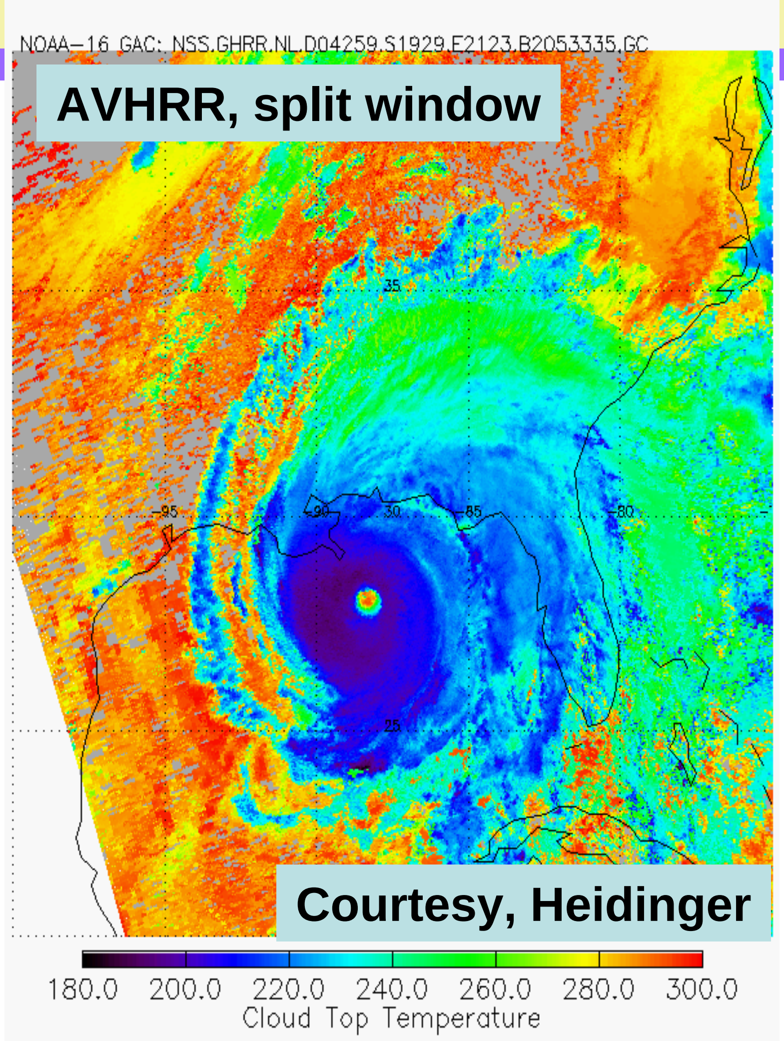
(Heidinger poster)

MODIS-AVHRR comparisons: Hurricane Ivan

Aqua, CO_2 slicing



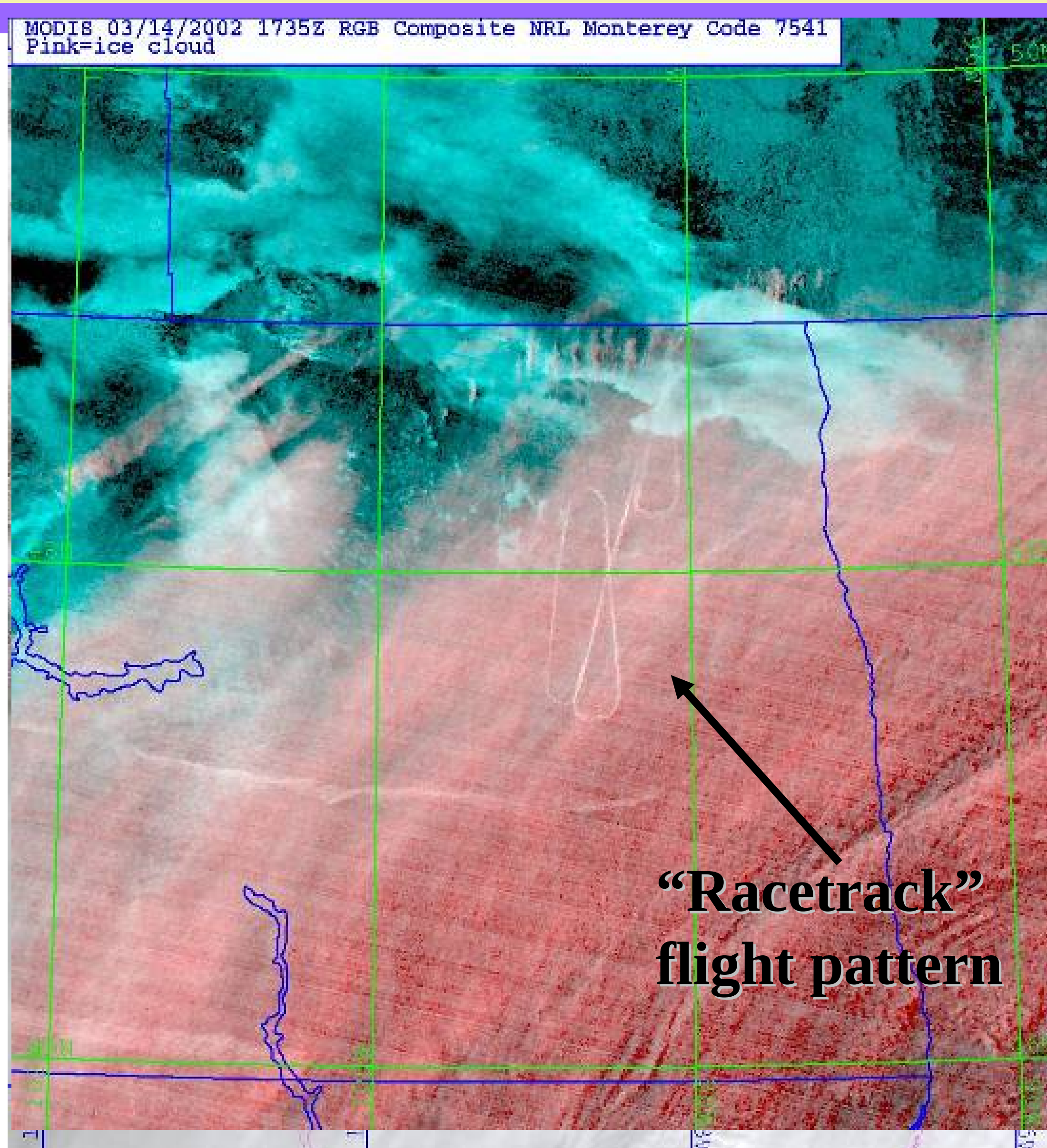
AVHRR, split window



Courtesy, Heidinger

We use different techniques based on the same physics (e.g. emission and scattering) for arriving at the same information

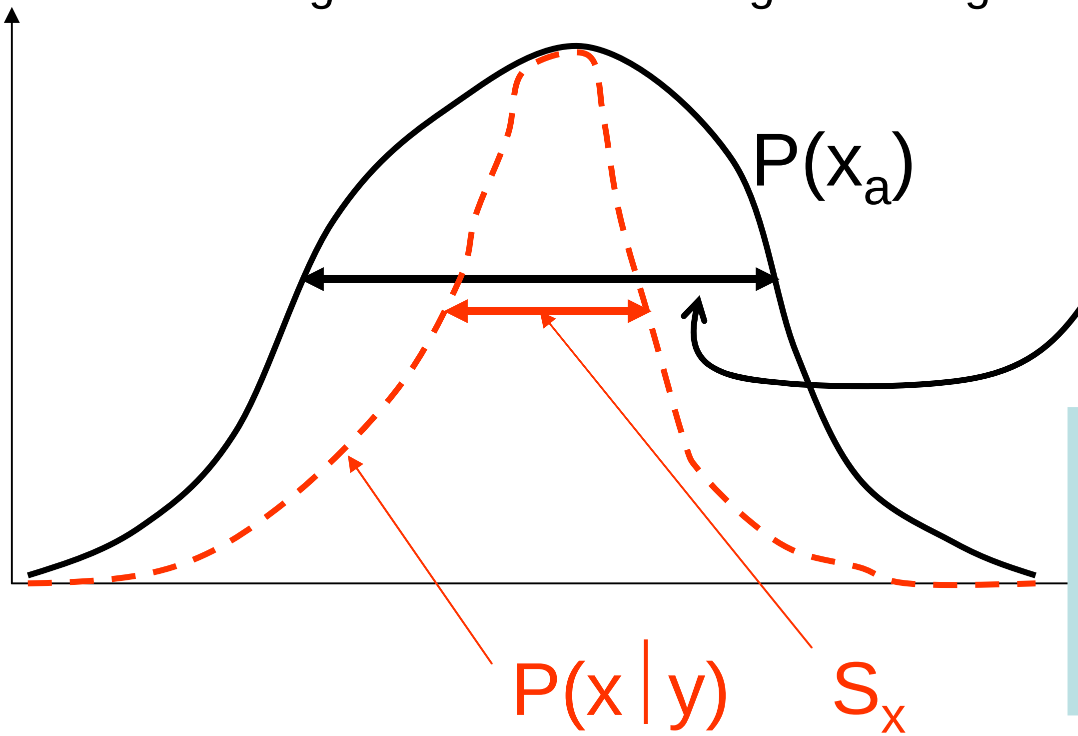
Example application: Aircraft Contrail Detection



Coutesy S. Miller

CloudSat Examples
1. Illustrating simple ideas of Information content

Information is an augmentation of existing knowledge



Therefore we might think that the measurement has added information if the 'volume' of the distribution is reduced

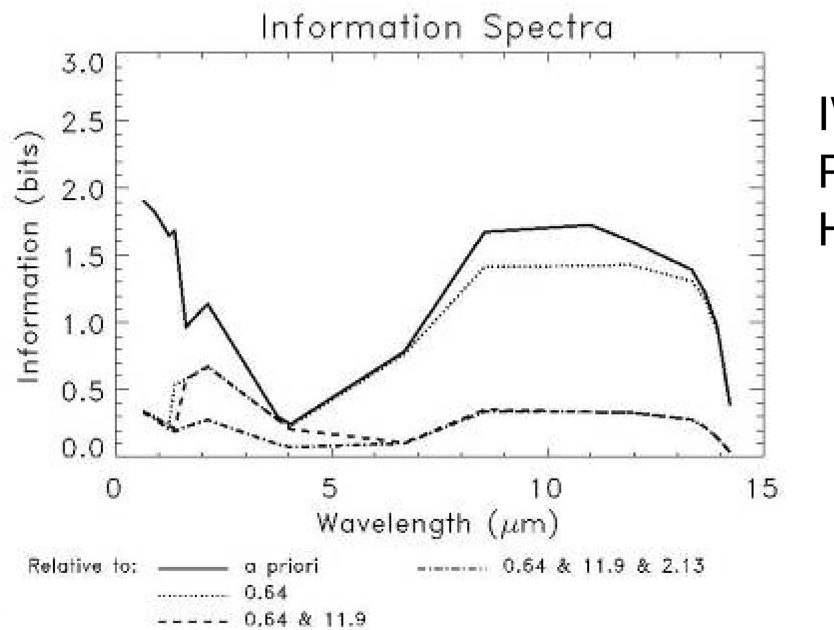
Shannon total information

$$H = s[P(x_a)] - s[P(x)]$$

The observing system identifies 2^H states over and above our background knowledge. It is a measure of system resolution.

MODIS ice cloud optical properties

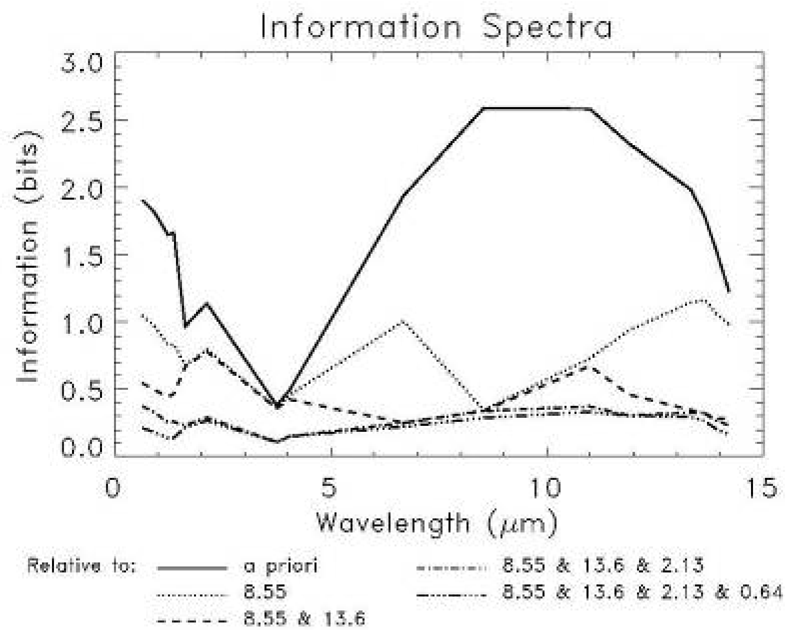
H$_i$



IWP~100
Re=16
Ht=9km

The point about this is there is no one optimal combination of channels – the combination of channels varies according to conditions

H$_i$

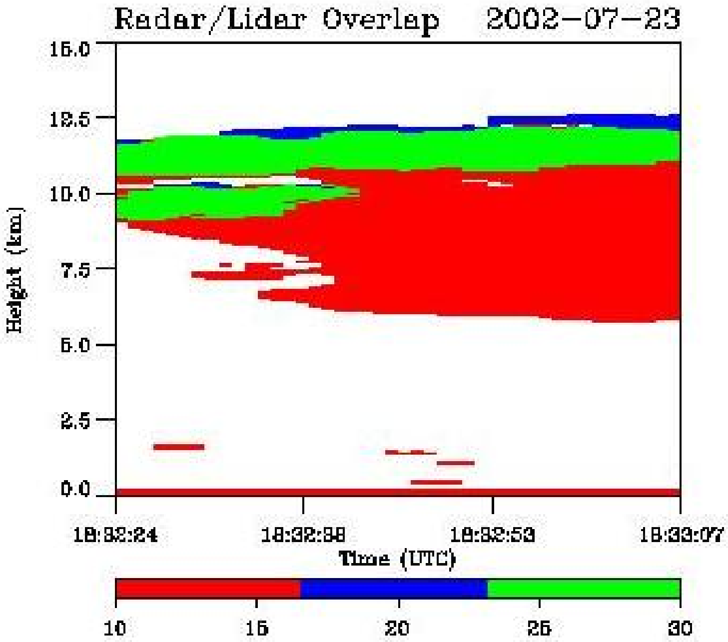


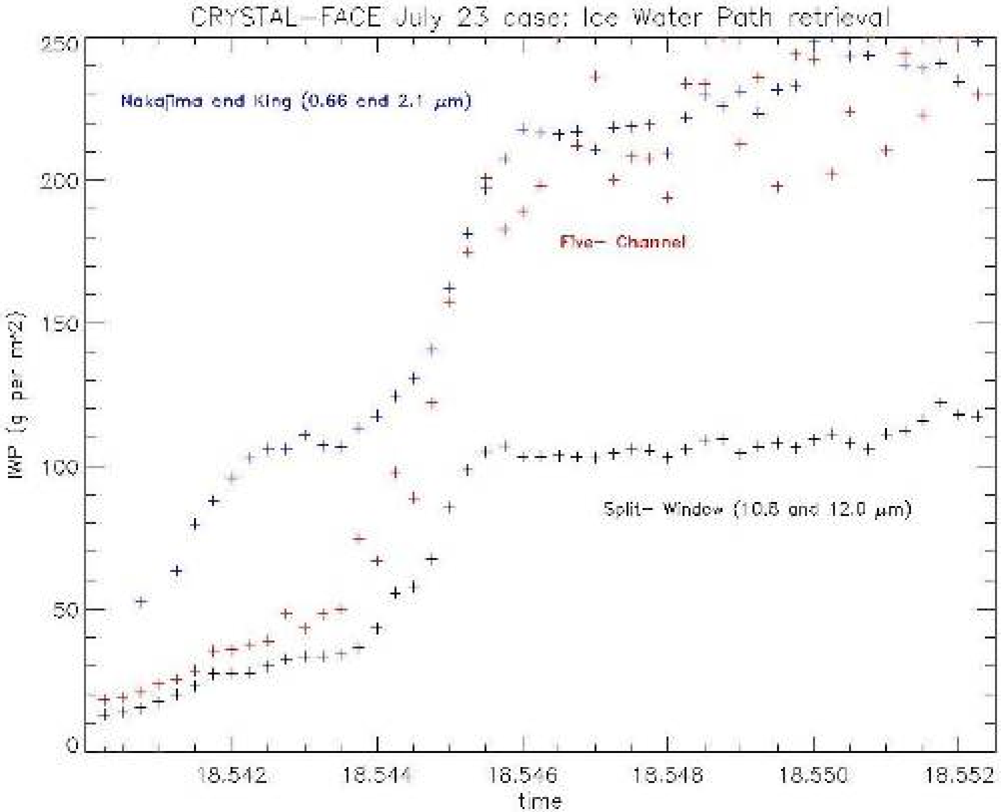
IWP~100
Re=16
Ht=14km

Cooper et al., 2004

Ice cloud Example - combining the physics of thermal emission and visible/nir scattering

Example using MAS data from Crystal FACE

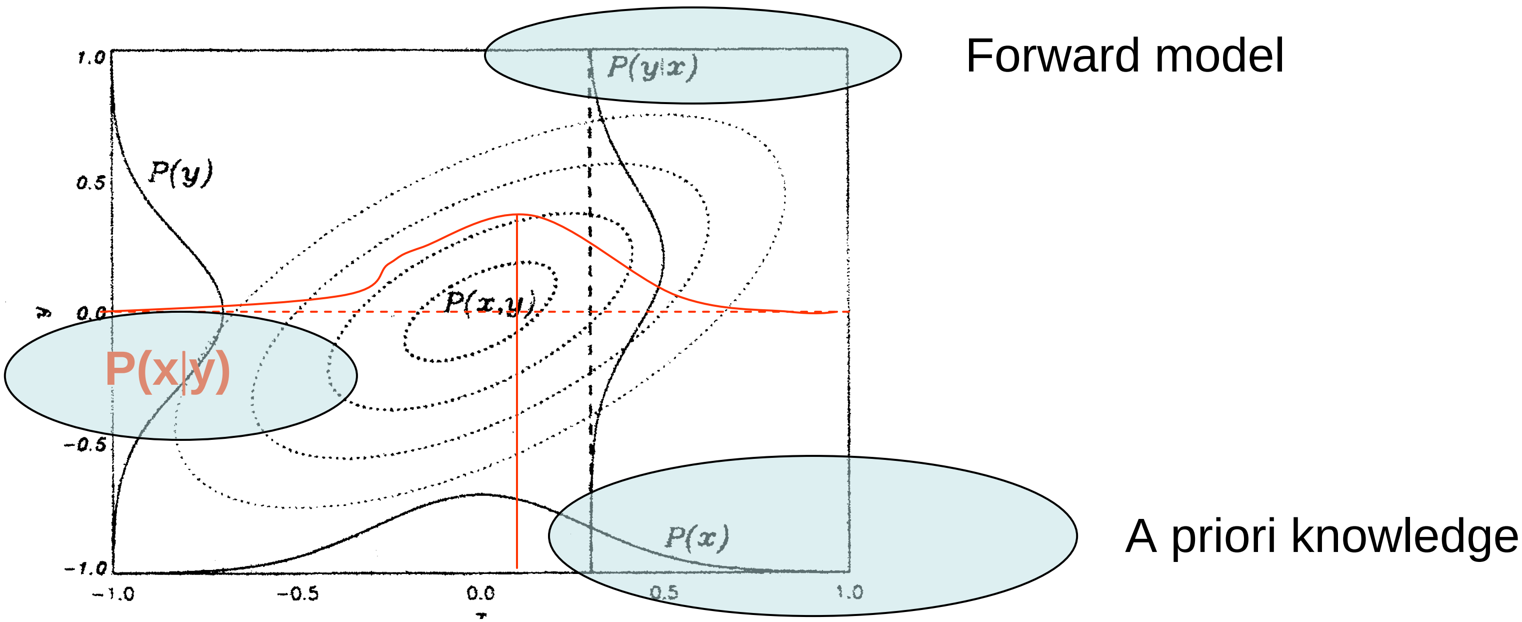




A 5 channel algorithm is being developed for CloudSat – this 5 channel method is superior to two channel methods currently being used to retrieve cirrus properties

Cooper et al., 2004

Adding measurements to some prior knowledge of the state: Bayes' Theorem



Desired (most likely) solution

Bayes Theorem

$$P(\mathbf{x}|\mathbf{y}) = \frac{P(\mathbf{y}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{y})}$$

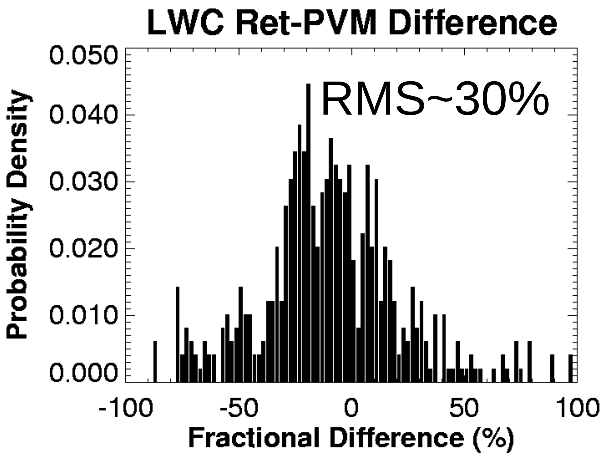
3. Error Validation

The CloudSat validation goal is to confirm the retrieval error estimates provided by all algorithms

- ground truth when possible (ISO GUM*, method A)

- component analyses (ISO GUM, method B)

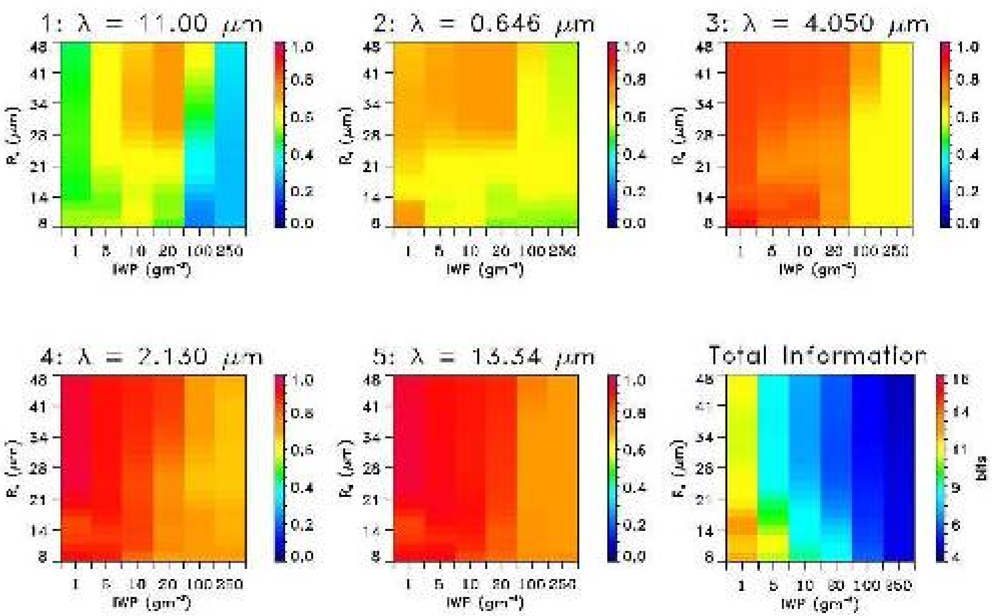
- consistency analyses (ISO GUM, method B)

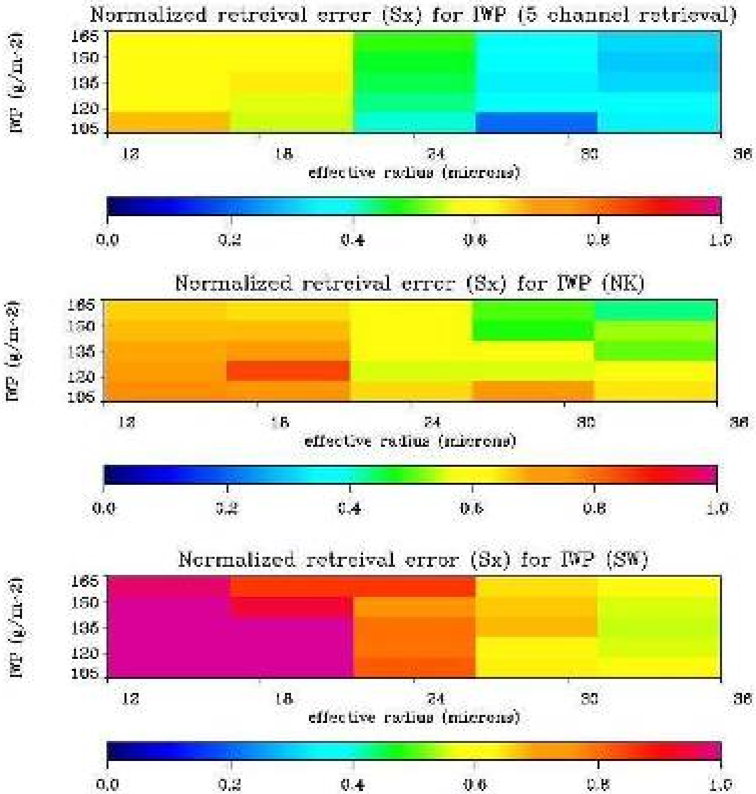


* *International Organization for Standards (ISO) Guide to the expression of uncertainty in Measurements*

Total errors derived from actual comparison of retrieved with in situ, method A

Ice cloud Example - combining the physics of thermal emission and visible/nir scattering





As we add channels, we can see how information is increased and how retrieval errors are reduced.

Passive:

Engelen and Stephens, 1997, JGR,6929-6939 (ozone)

Heidinger and Stephens, 1998; 2000,J.Atmos.Sci.,57,(cloud)

Miller, Austin and Stephens, 2001,JGR,106,17981-17995 (cloud)

Cooper, L'Ecuyer and Stephens,2003, JGR,108,(cloud)

Engelen et al., 2002; CO_2

Passive-Passive

Engelen and Stephens,1999;QJRMS,125,331-351; water vapor

Christi and Stephens, 2004;JGR; CO_2

Active - Passive:

Stephens, Engelen, Vaughan and Anderson,2001,JGR, (aerosol/cloud)

Austin and Stephens, 2001, *JGR*, 106, 28,233 - 28,242) (cloud).

L'Ecuyer and Stephens, 2002, J.Appl. Met., 41,271-285 (precip).

Benedetti, Stephens and Haynes, 2003; JGR, 108 (cirrus)

Austin and Stephens, 2004; JGR submitted (cloud)

Mitrescu, Haynes,Stephens, Heymsfield and McGill, 2004 (cirrus)

Information Content:

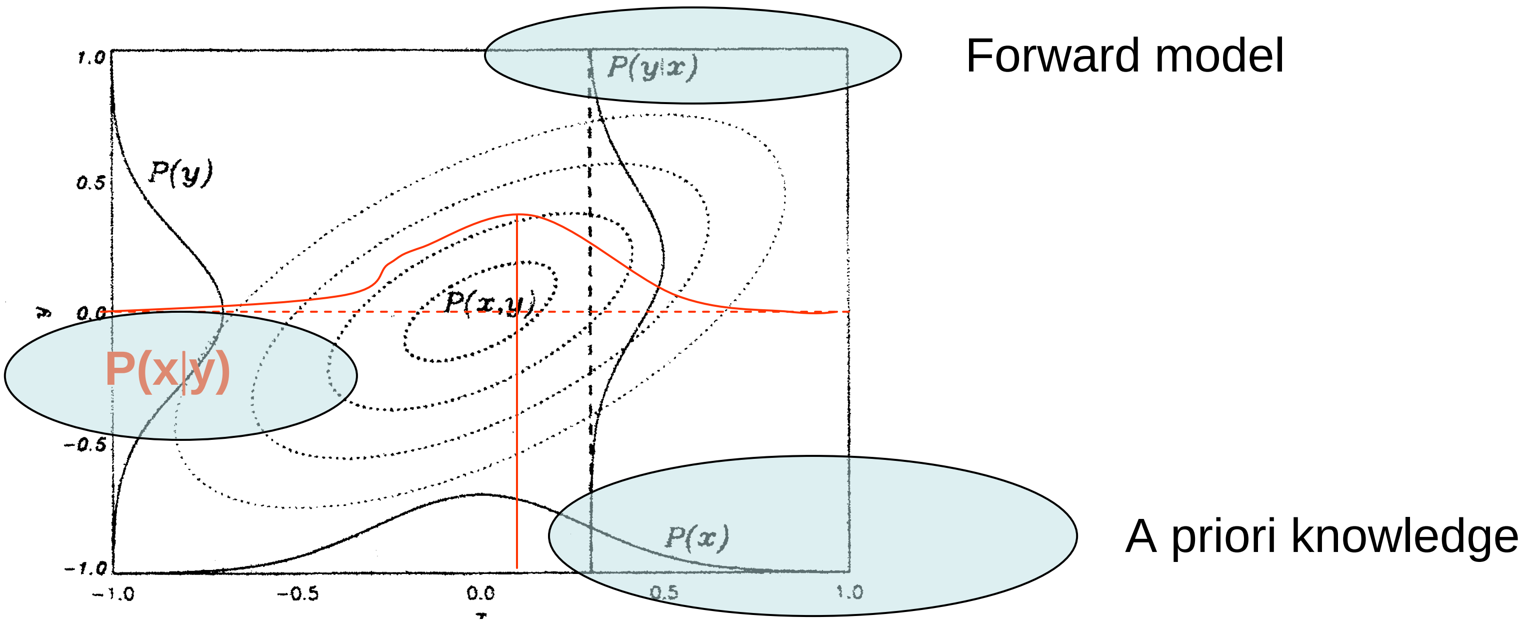
Engelen and Stephens, 2003, J.Appl.Met.

L'Ecuyer, Cooper, Leesman,,Stephens, 2004; In preparation.

Cooper, et al., 2004; in preparation

Labonnote and Stephens, 2004;JGR

Adding measurements to some prior knowledge of the state: Bayes' Theorem



Desired (most likely) solution

Bayes Theorem

$$P(\mathbf{x}|\mathbf{y}) = \frac{P(\mathbf{y}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{y})}$$

Example: Return to our 'simple' example and apply Optimal Estimation technique

A priori assumption $\hat{\mathbf{x}}_a = \begin{pmatrix} 1.2 \\ 1.1 \end{pmatrix}$

Assume diagonal covariance matrices with 0.001 for the error in the measurements and 0.5 for the error in the *a priori* guess.

$$\hat{\mathbf{x}} = \left(\mathbf{K}^T \mathbf{S}_y^{-1} \mathbf{K} + \mathbf{S}_a^{-1}\right)^{-1} \left(\mathbf{K}^T \mathbf{S}_y^{-1} \mathbf{y} + \mathbf{S}_a^{-1} \mathbf{x}_a\right) = \begin{pmatrix} 1.05 \\ 0.95 \end{pmatrix}$$

We also obtain a covariance matrix for the result:

$$\mathbf{S}_x = \left(\mathbf{K}^T \mathbf{S}_y^{-1} \mathbf{K} + \mathbf{S}_a^{-1}\right)^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.25 & 0.25 \end{pmatrix}$$

So what have we gained???

The CloudSat Liquid Water content example

$$N(r) = \frac{N_T}{\sqrt{2\pi}\sigma_{log} r} \exp\left[\frac{-\ln^2(r/r_g)}{2\sigma_{log}^2}\right]$$

Assume N_T and σ_{log} are constant in height

$$\left.\begin{aligned} Z_{dBZ}(z_i) &= 10\log[64 N_T r_{gi}^6 \exp(18\sigma_{\log}^2)] \\ \tau &= \sum_{i=1}^{p} 2\pi N_T r_{gi}^2 \exp(2\sigma_{\log}^2)\Delta z \end{aligned}\right\}$$

"forward model" f(x,b)

Measurements vector

$$\mathbf{y} = \begin{bmatrix} Z'_{dBZ}(z_1) \\ \vdots \\ Z'_{dBZ}(z_p) \\ \tau \end{bmatrix}$$

m = p+1 elements

State vector

$$\mathbf{x} = \begin{bmatrix} r_g(z_1) \\ \vdots \\ r_g(z_p) \\ N_T \\ \sigma_{\log} \end{bmatrix}$$

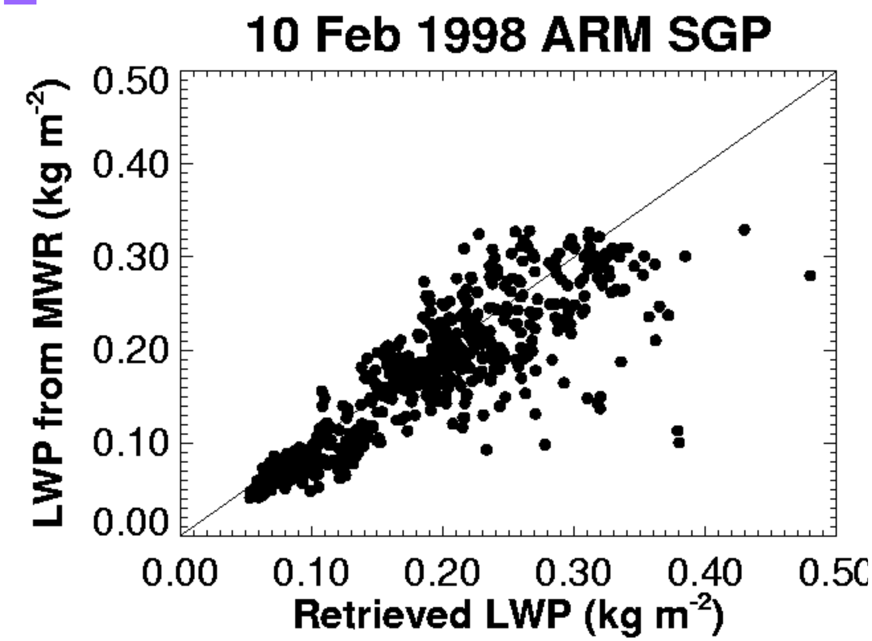
n = p+2 elements

A priori vector

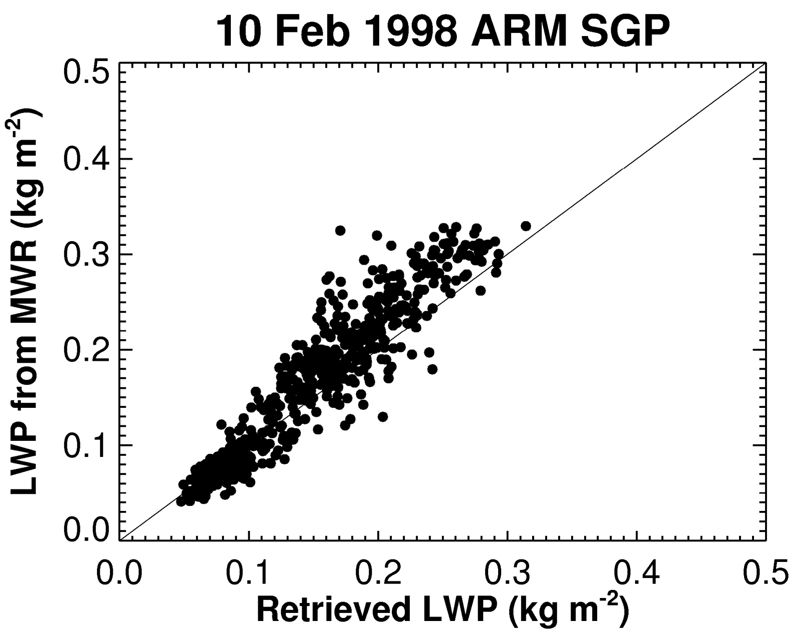
$$\mathbf{x_a} = \begin{bmatrix} r_{ga}(z_1) \\ \vdots \\ r_{ga}(z_p) \\ N_{Ta} \\ \sigma_{\log_a} \end{bmatrix}$$

p+2 elements

Application to ARM data



Old with width parameter specified



New with width parameter retrieved

Passive:

Engelen and Stephens, 1997, JGR,6929-6939 (ozone)
Heidinger and Stephens, 1998; 2000,J.Atmos.Sci.,57,(cloud)
Miller, Austin and Stephens, 2001,JGR,106,17981-17995 (cloud)
Cooper, L'Ecuyer and Stephens,2003, JGR,108,(cloud)
Engelen et al., 2002; CO_2

Passive-Passive

Engelen and Stephens,1999;QJRMS,125,331-351; water vapor
Christi and Stephens, 2004;JGR; CO_2

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Austin and Stephens, 2001, *JGR*, 106, 28,233 - 28,242) (cloud).
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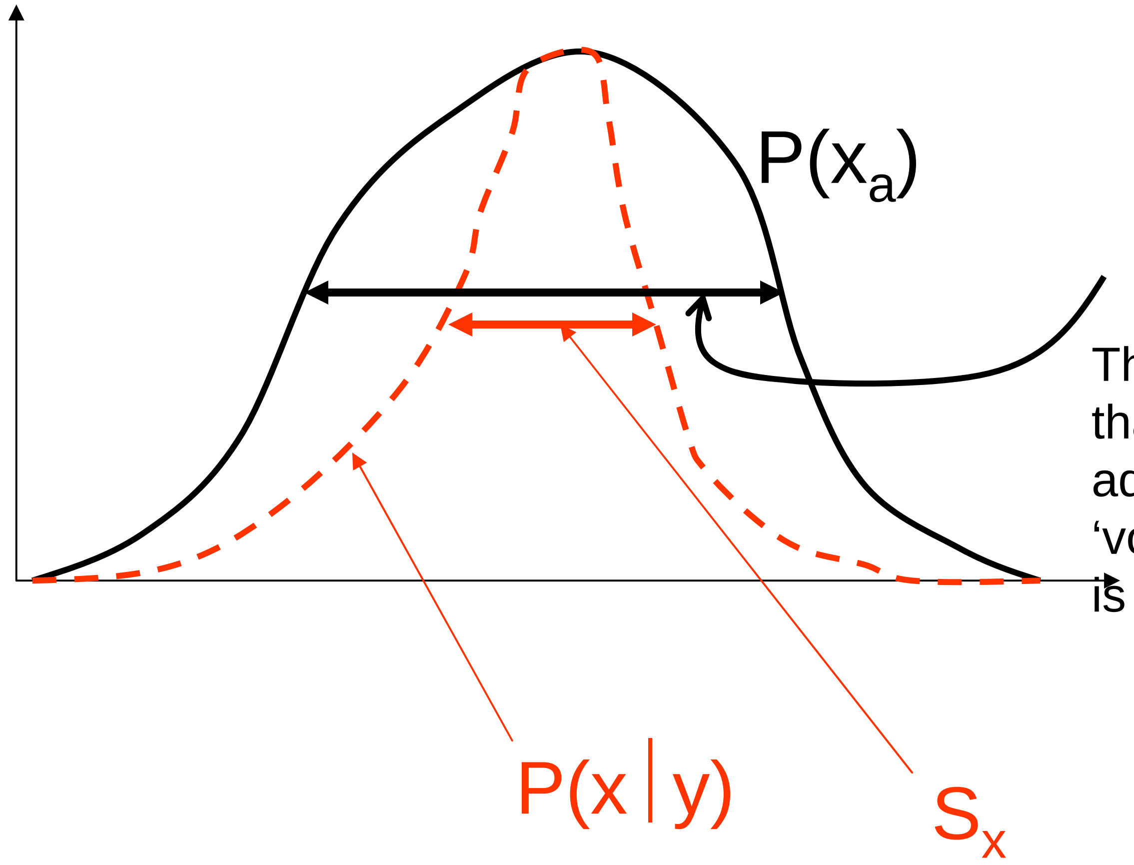
L'Ecuyer, Cooper, Leesman,,Stephens, 2004; In preparation.
Cooper, et al., 2004; in preparation
Labonnote and Stephens, 2004;JGR

Information content: elementary ideas

Information is an augmentation of existing knowledge thus it is a relative concept



Therefore we might think that the measurement has added information if the 'volume' of the distribution is reduced

Shannon's measure of information

Entropy is a measure of the # of distinct states of a system, and thus a measure of information about that system. If the system is defined by the pdf P(x), then

$$s(P) = -k\int P(x)\ln P(x)dx$$

for

$$\mathrm{P(x)} \rightarrow \mathrm{exp[-(x-}\langle \mathrm{x} \rangle)^T S_x \mathrm{(x-}\langle \mathrm{x} \rangle))]$$

$$s(P) = \frac{1}{2}\ln S_x$$

In our context, information is the change (reduction) in entropy of the 'system' after a measurement is made

$$H = s(P(x_a)) - s(P(x))$$

$$H = \frac{1}{2}\ln\left|S_a S_x^{-1}\right|$$

Summary of information properties

Property	Interpretation
A	**Provides a measure of where information comes to produce the retrieved state x**
H	**The observing system identifies 2H states over and above our background knowledge. It is a measure of system resolution.**
$dfs = Tr(I - S_a S_x^{-1})$	**# of measurements above noise**
$\tilde{K} = S_y^{-1/2} K S_a^{1/2}$ $K_{ij} = \dfrac{\partial f_i}{\partial x_j}$	**Singular values of this scaled Jacobean matrix above unity tell us about how many pieces of information are contained in the measurements. The singular vectors tell us what combination of state parameters are retrievable**