Radiative Transfer in the Earth Atmosphere: The Fundamentals

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2009 Update

Acknowledgements to Dr. M.D King, University of Colorado
& Professor G. Kattawar, Texas A&M University

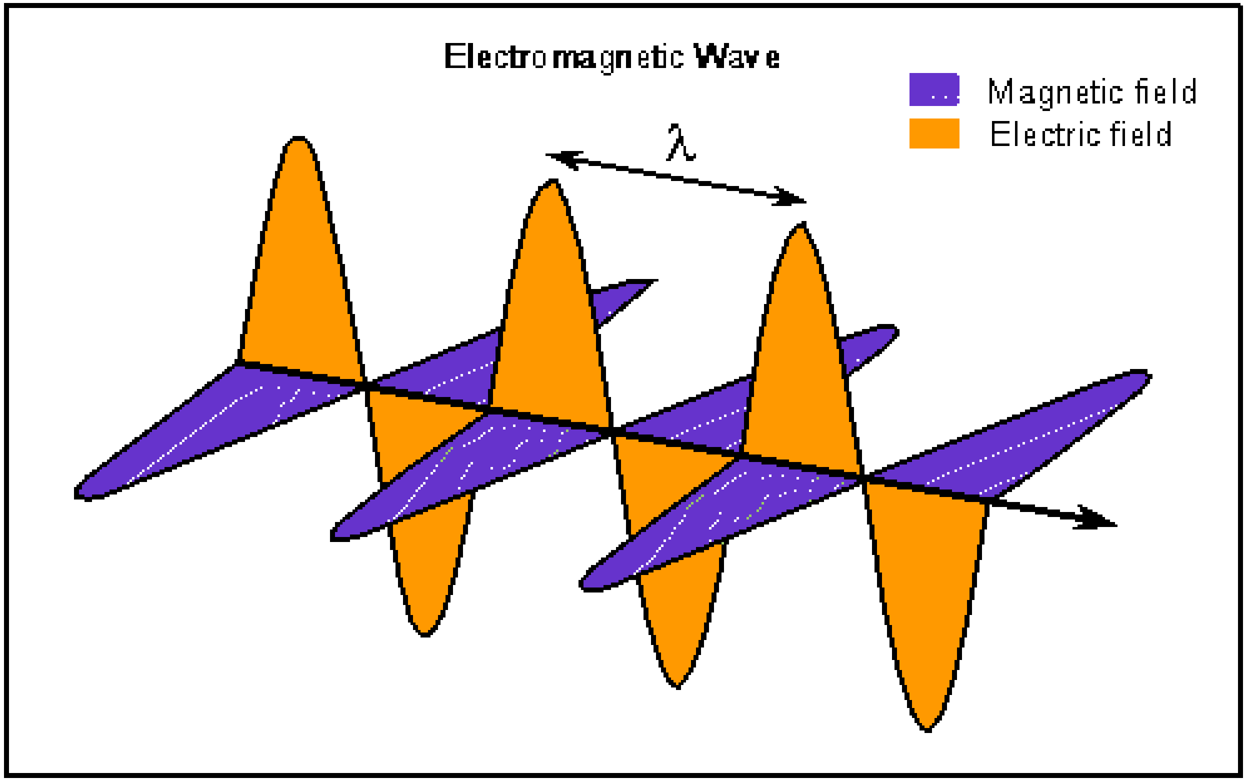
Outline

1. Electromagnetic energy interactions
2. Electromagnetic radiation models (wave/particle)
3. Atmospheric scattering
4. Atmospheric absorption
5. Radiometric quantities
6. Summary

1. Electromagnetic Energy Interactions

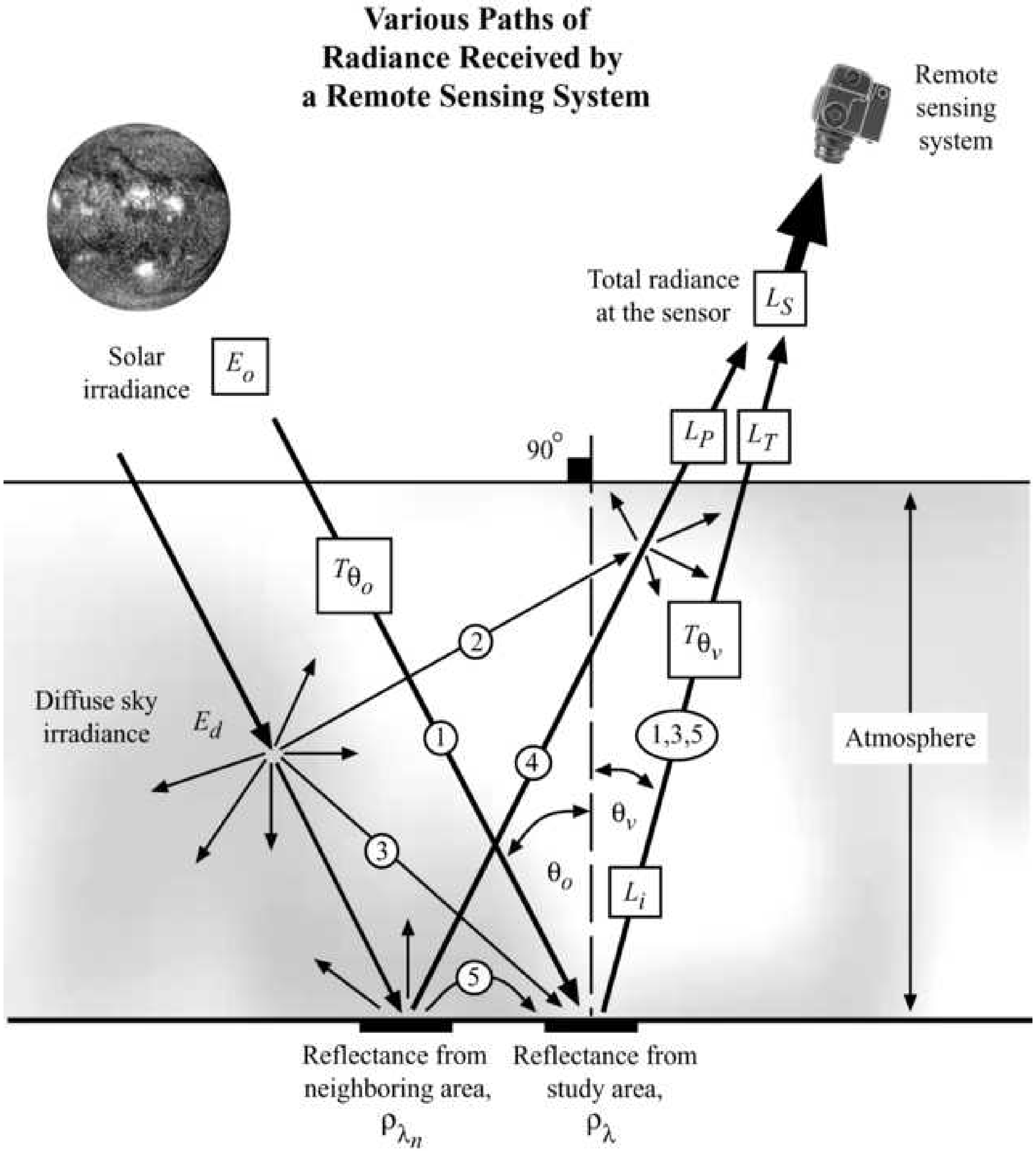
1. Electromagnetic Energy Interactions

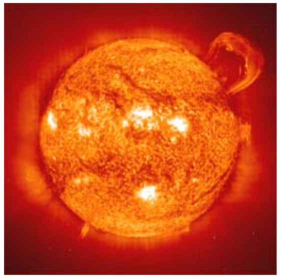
- When the energy being remotely sensed comes from the Sun, the energy:
  - Propagates through the vacuum of space
  - Interacts with the Earth's atmosphere, surface, and atmosphere
  - Reaches the remote sensor (interacts with various optical systems, filters, emulsions, or detectors)



$$C=3*10^8=v * \lambda$$

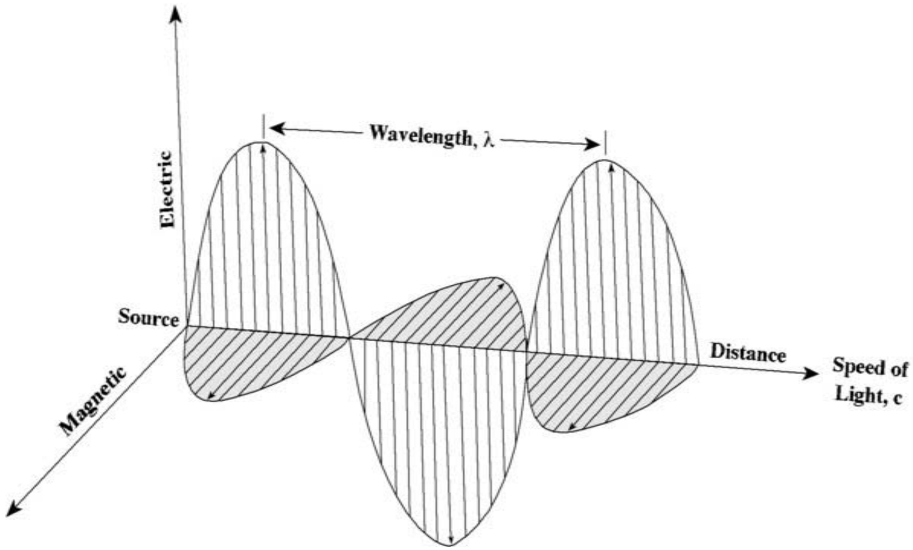
- Two fields:
  - Electrical & magnetic
- Travel perpendicular & speed of light
- Property & behaves in predictable way
- Frequency & wavelength
- Photons/quanta





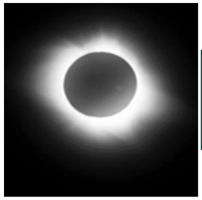
Energy-matter interactions in the atmosphere, at the study area, and at the remote sensor detector

2. Electromagnetic Radiation Models



The Wave Model of Electromagnetic Energy

- Frequency: the number of wavelengths that pass a point per unit time
- Wavelength: the mean distance between maximums (or minimums)
- Common units: micrometers (μm) or nanometers (nm)
- One cycle per second is termed one hertz (1 Hz)

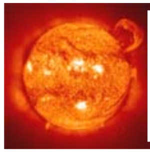


Wave Model of Electromagnetic Energy

The relationship between the wavelength, λ, and frequency, ν, of electromagnetic radiation is based on the following formula, where *c* is the speed of light:

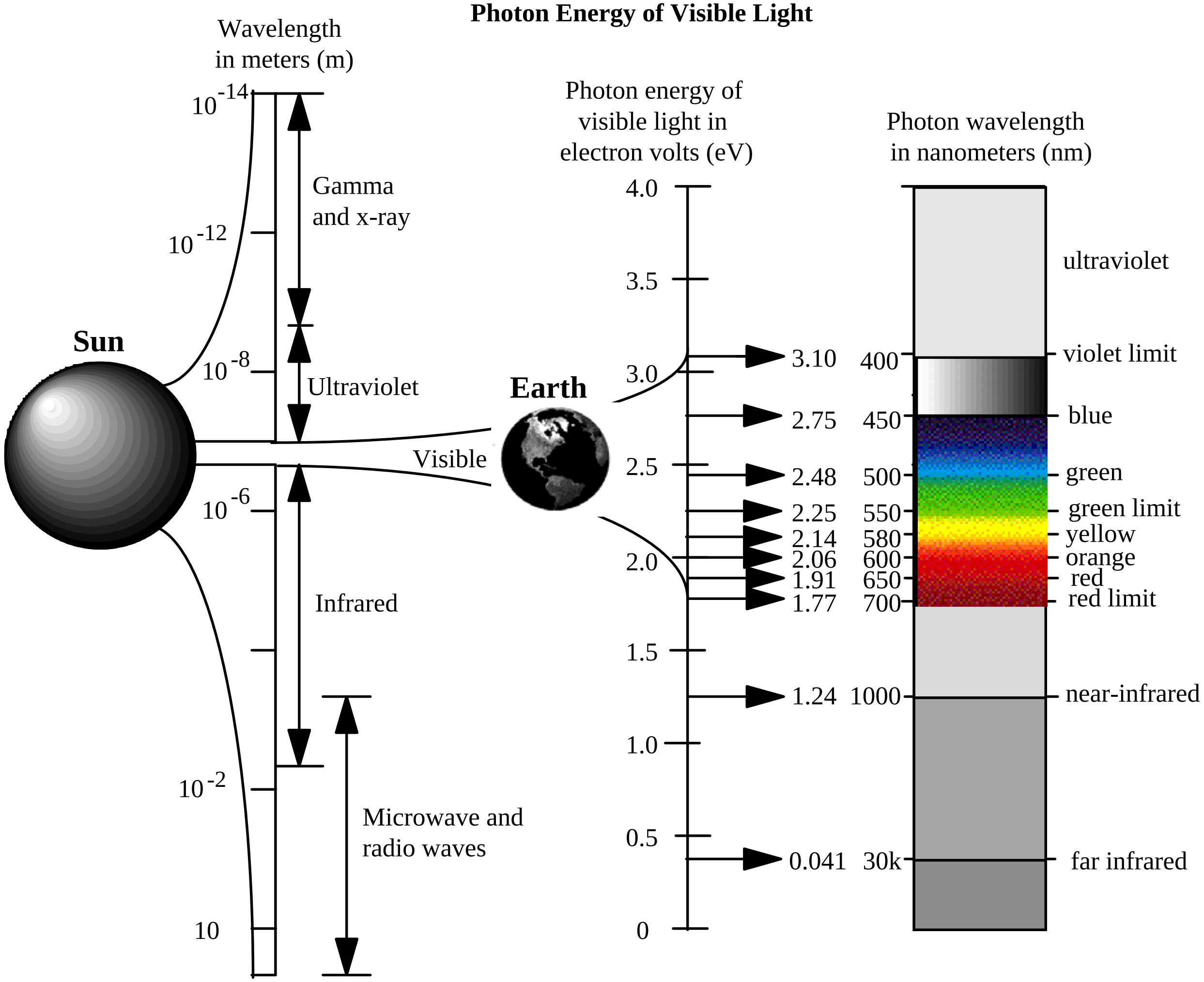
$$c = \lambda \cdot v \qquad v = \frac{c}{\lambda} \qquad \lambda = \frac{v}{c}$$

Note that frequency, ν, is inversely proportional to wavelength, λ
The longer the wavelength, the lower the frequency, and vice-versa



Electromagnetic (EM) Spectrum

Electromagnetic Spectrum and the Photon Energy of Visible Light



The Sun produces a *continuous spectrum* of energy from gamma rays to radio waves that continually bathe the Earth in energy

The visible portion of the spectrum may be measured using wavelength (measured in μm or nm) or electron volts (eV)

- All units are interchangeable

Stefan-Boltzmann Law

- The total emitted radiation (M_λ) from a blackbody is proportional to the fourth power of its absolute temperature
  - This is known as the Stefan-Boltzmann law and is expressed as:

$$M_\lambda = \sigma T^4$$

where σ is the Stefan-Boltzmann constant = 5.6697 x 10^{-8} W m^{-2} K^{-4}
  - T = absolute temperature (in Kelvin)
- The greater the T, the greater the amount of radiant energy exiting the object
  - The temperature 0˚C (in the common Celsius scale) corresponds to 273 K

Wien's Displacement Law

- To compute its dominant wavelength (λ_{max}) as:

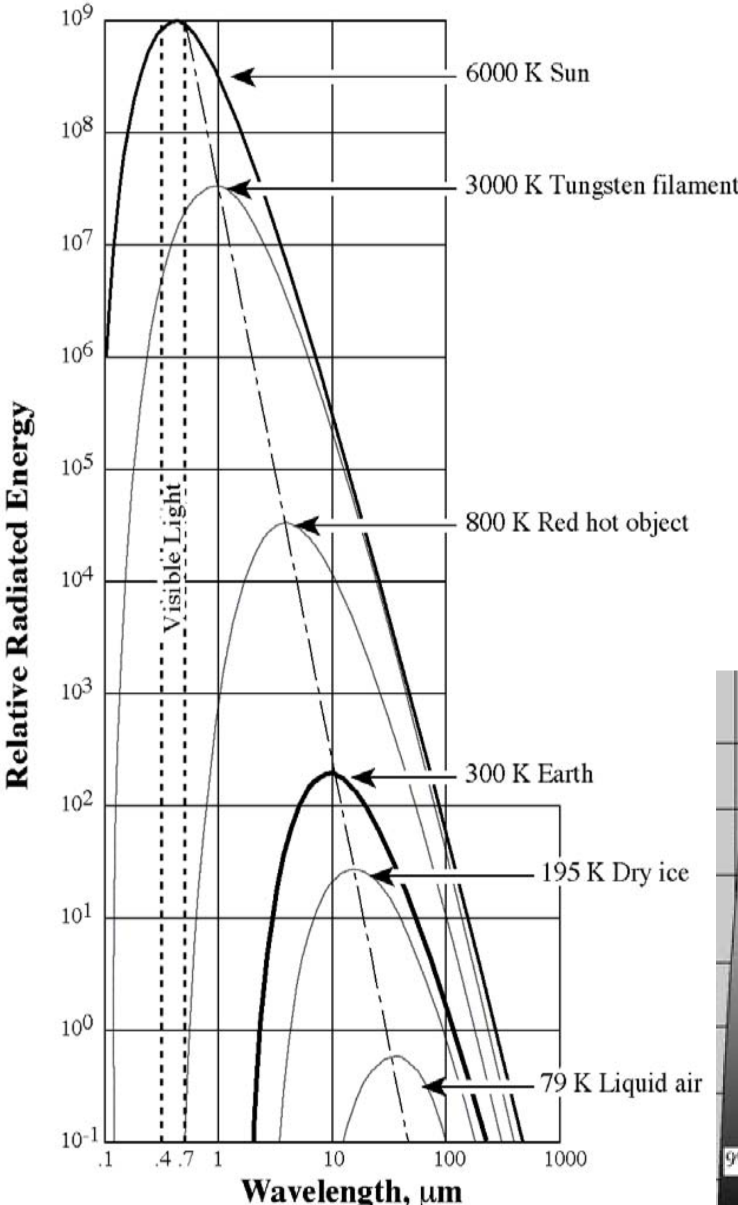
$$\lambda_{max} = k / T$$

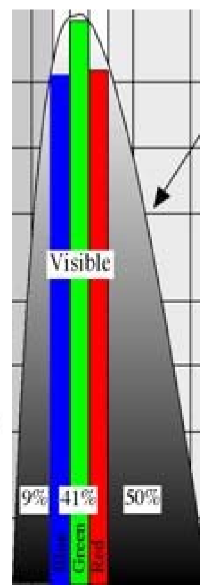
  where *k* is a constant equaling 2898 μm K, and *T* is temperature in degrees Kelvin

- The Sun approximates a 6,000 K blackbody, therefore its dominant wavelength is:

$$0.483\ \mu m = 2898\ \mu m\ K / 6000\ K$$

  - *T* determines the wavelength
- Therefore from the (λ_{max}) information, *T* can be calculated

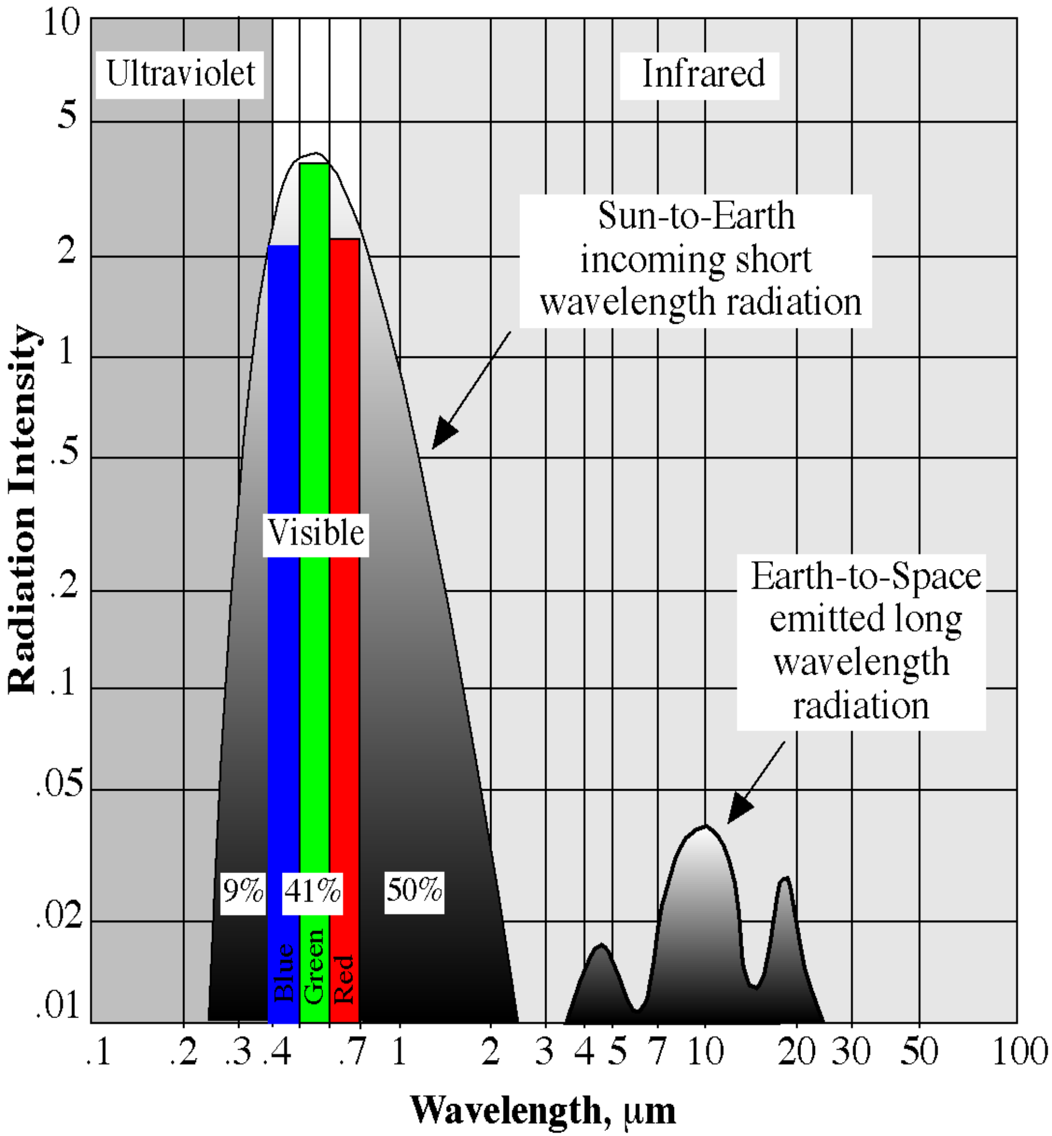




Blackbody Radiation Curves

- Blackbody radiation curves for the Sun: temperature approximate 6,000 K
- For Earth: 300 K
- As the temperature of the object increases, its dominant wavelength shifts toward the short wavelength portion of the spectrum

Radiant Intensity of the Sun



- The Sun (6,000 K blackbody) dominant: 0.5 µm
- Earth (300 K blackbody)
- Dominant: 9.7 µm
- Sun: 41%: visible region from 0.4 - 0.7 µm
  - The other 59% (<0.4 µm) and (>0.7 µm)
- Eyes are only sensitive to light from the 0.4 to 0.7 µm
- Remote sensor detectors can be made sensitive to energy in the non-visible regions of the spectrum

3. Atmospheric Scattering

Atmospheric Scattering

Electromagnetic radiation is propagated through the earth's atmosphere almost at the speed of light in a vacuum

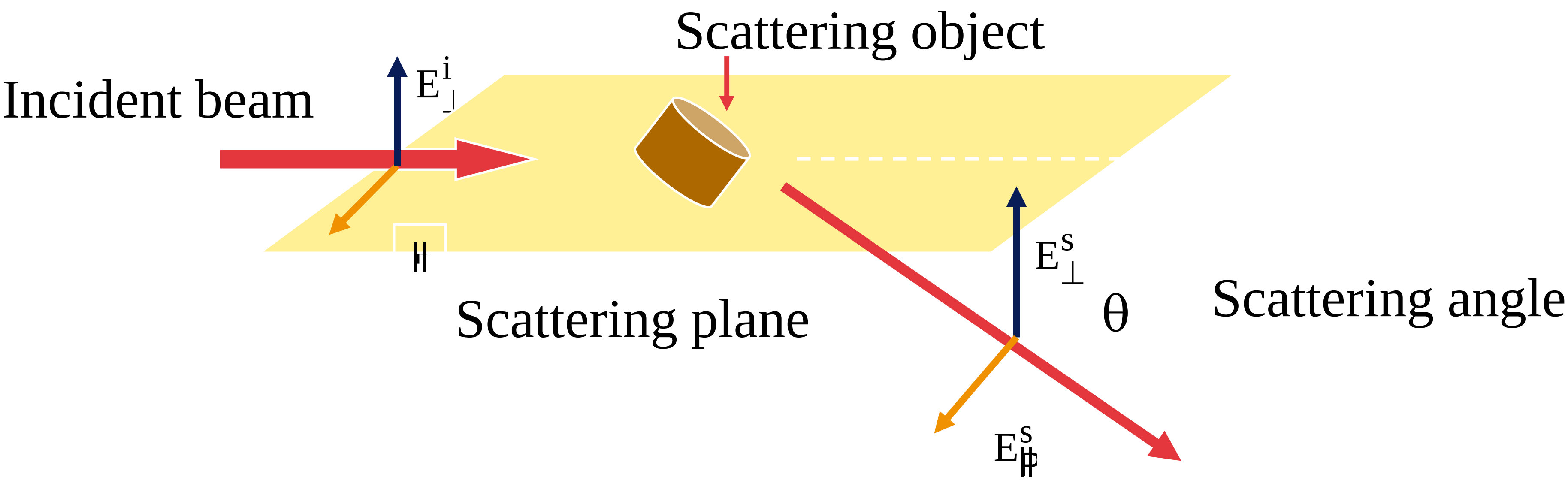
- Unlike a vacuum in which nothing happens, however, the atmosphere may affect
  - speed of radiation
  - wavelength
  - Intensity
  - Polarization
  - spectral distribution
  - direction

Atmospheric Scattering

The type of scattering is a function of:

- The wavelength of the incident radiant energy
- The size of the gas molecule, dust particle, or cloud and rain droplet encountered
- The phase of clouds

Scattering Geometry



Scattering amplitude matrix

$$\begin{pmatrix} E^s_\parallel \\ E^s_\perp \end{pmatrix} = \frac{e^{ikr}}{-ikr} \begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} \begin{pmatrix} E^i_\parallel \\ E^i_\perp \end{pmatrix}$$

Volume Scattering and Extinction Coefficient

- Volume scattering coefficient [σ_{sca}]
  - Fractional amount of energy scattered in all directions per unit length of transit [m^{-1}]

$$\sigma_{sca} = \int S(\Theta)d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} S(\Theta)\sin\Theta d\Theta d\phi$$

- Volume absorption coefficient [σ_{abs}]
  - Fractional amount of energy absorbed per unit length of transit [m^{-1}]
- Volume extinction coefficient [σ_{ext}]
  - Fractional amount of energy attenuated per unit length of transit [m^{-1}]

$$\sigma_{ext} = \sigma_{sca} + \sigma_{abs}$$

- Single scattering albedo [ω_0]
  - Fraction of energy scattered to that attenuated

$$\omega_0 = \sigma_{sca}/(\sigma_{sca} + \sigma_{abs})$$

Optical Thickness

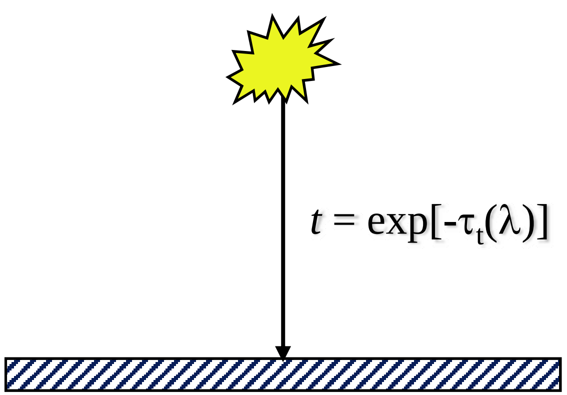
- Optical depth [τ]
  - Total attenuation along a path length, generally a function of wavelength [dimensionless]

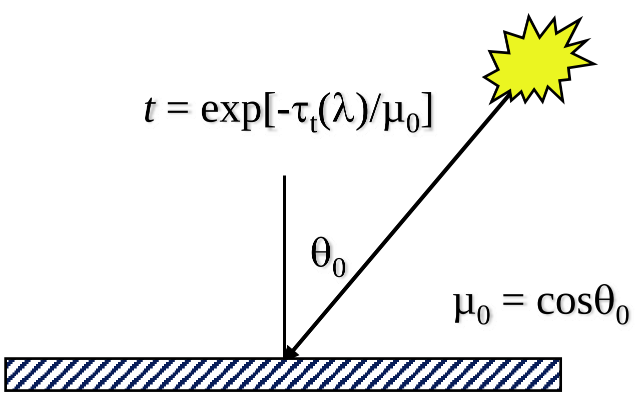
$$\tau(\lambda) = \int_0^X \sigma_{ext} dx$$

- Total optical thickness of the atmosphere [τ_t]
  - Total attenuation in a vertical path from the top of the atmosphere down to the surface

$$\tau_t(\lambda) = \int_0^\infty \sigma_{ext} dz$$

- Transmission of the direct solar beam





Scattering Phase Function

- Scattering phase function is defined as the ratio of the energy scattering per unit solid angle into a particular direction to the average energy scattered per unit solid angle into all directions

$$P(\cos\Theta) = \frac{S(\Theta)}{\frac{\int S(\Theta)dW}{4\pi}} = \frac{4\pi\, S(\Theta)}{\sigma_{sca}}$$

with this definition, the phase function obeys the following normalization

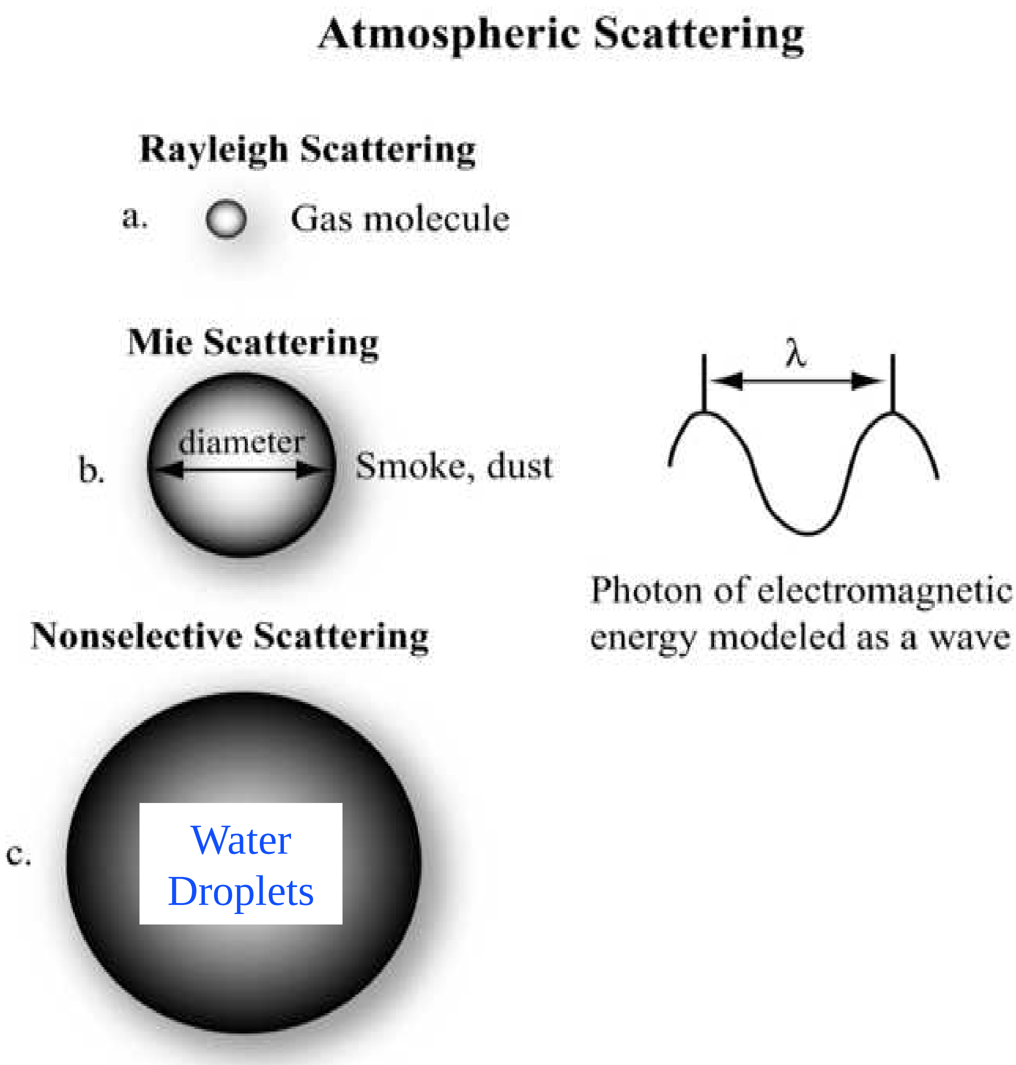
$$1 = \frac{1}{4\pi}\int_{0}^{4\pi} P(\cos\Theta)d\Omega$$

$$= \frac{1}{2}\int_{-1}^{1} P(\cos\Theta)d\cos\Theta$$

- Rayleigh (molecular) scattering phase function

$$P(\cos\Theta) = \frac{3}{4}(1 + \cos^2\Theta)$$

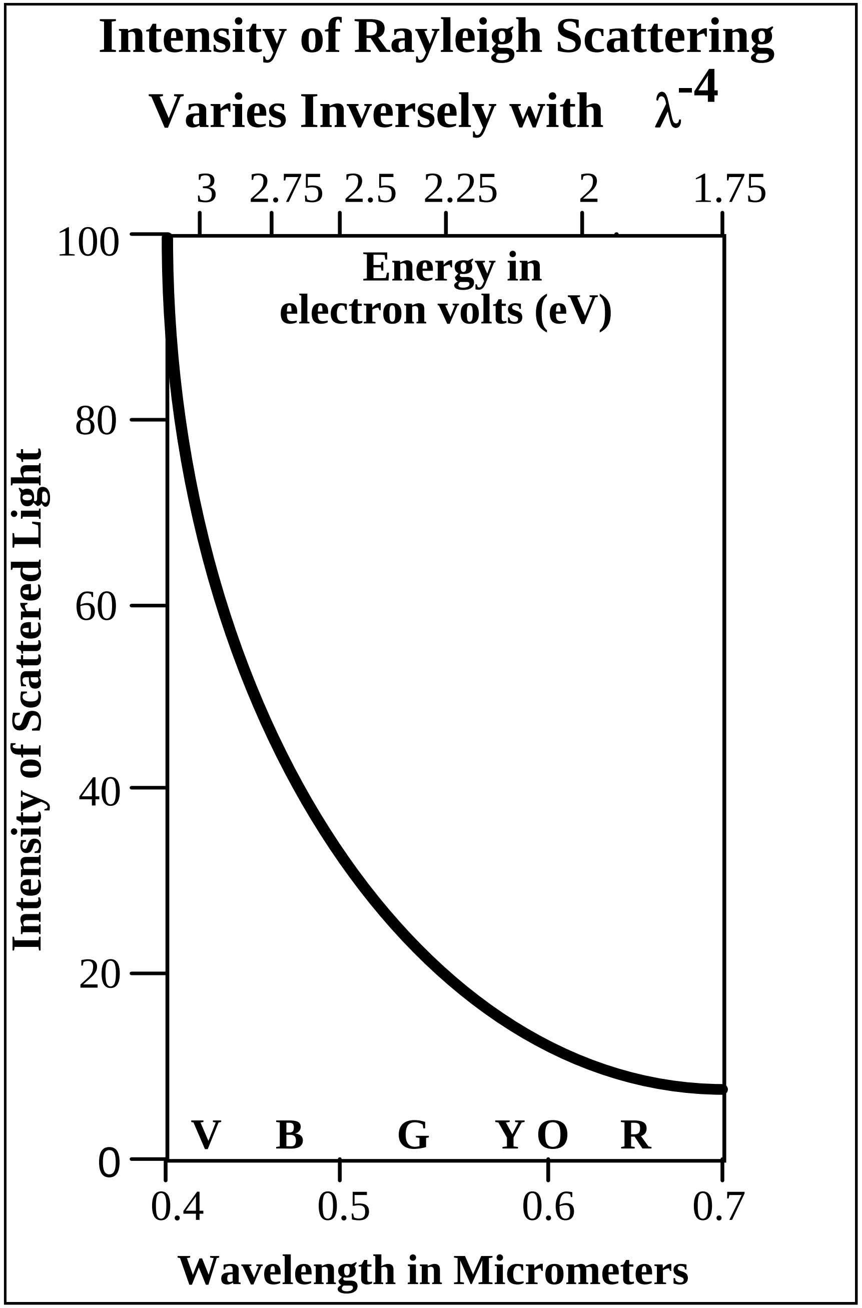
Atmospheric Scattering



Reflection: the direction predictable
Scattering: direction unpredictable

Based on wavelength of incident radiant energy, the size of the gas molecule, dust particle, or water vapor droplet essentially three types of scattering:

- Rayleigh
- Mie
- non-selective scattering



Rayleigh Scattering

- The amount of scattering is inversely related to the fourth power of the radiation's wavelength (λ^{-4})
- For example, blue light (0.4 μm) is scattered 16 times more than near-infrared light (0.8 μm)

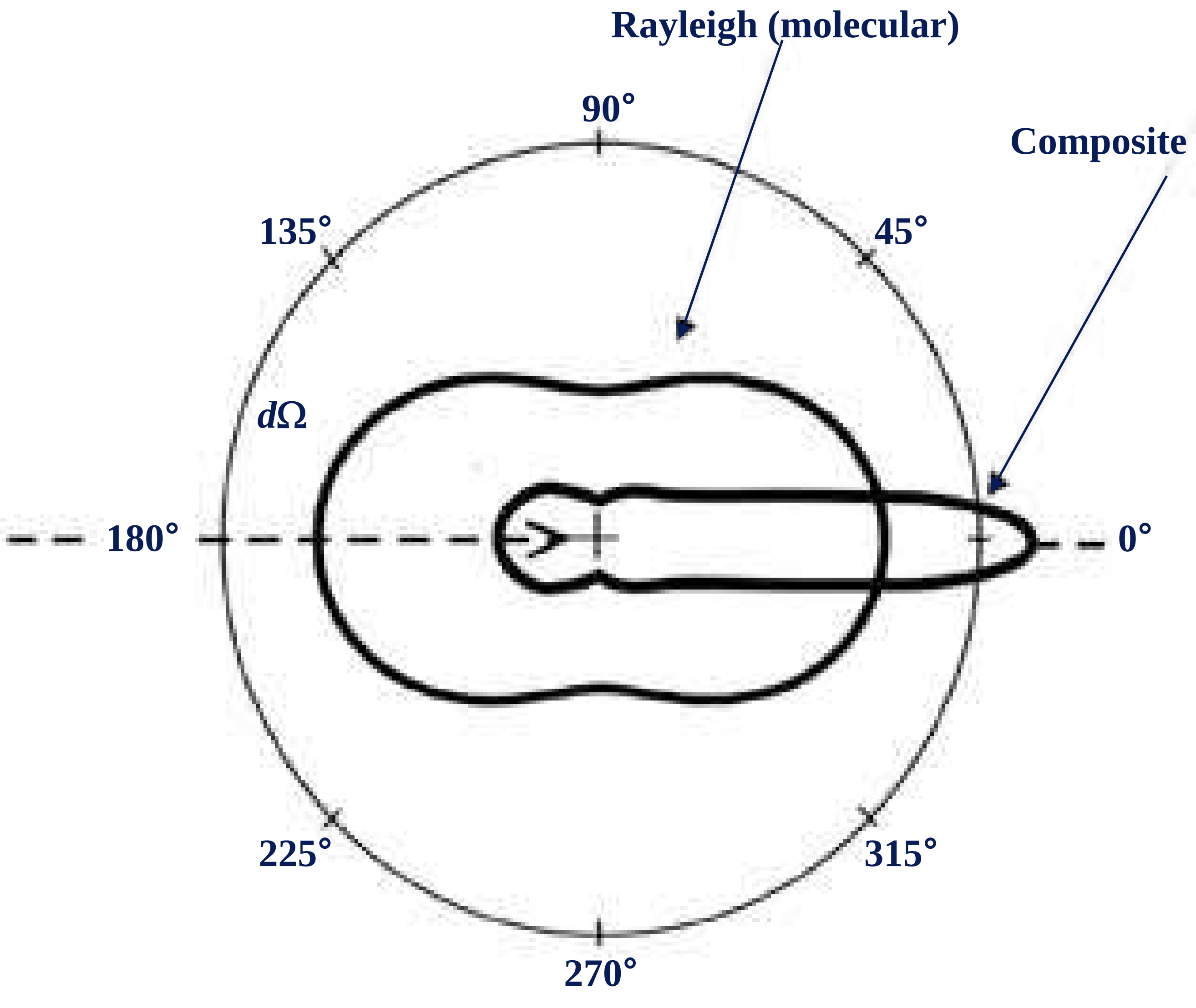
Mie Scattering

- Mie scattering: when essentially spherical particles present in the atmosphere with diameters approximately equal to the wavelength of radiation
- For visible light, water droplets, dust, and other particles ranging from a few tenths of a micrometer to several micrometers in diameter are the main scattering agents
  - The amount of scatter is greater than Rayleigh scatter and the wavelengths scattered are longer
- Pollution also contributes to beautiful sunsets and sunrises
  - The greater the amount of smoke and dust particles in the atmospheric column, the more violet and blue light will be scattered away and only the longer orange and red wavelength light will reach our eyes

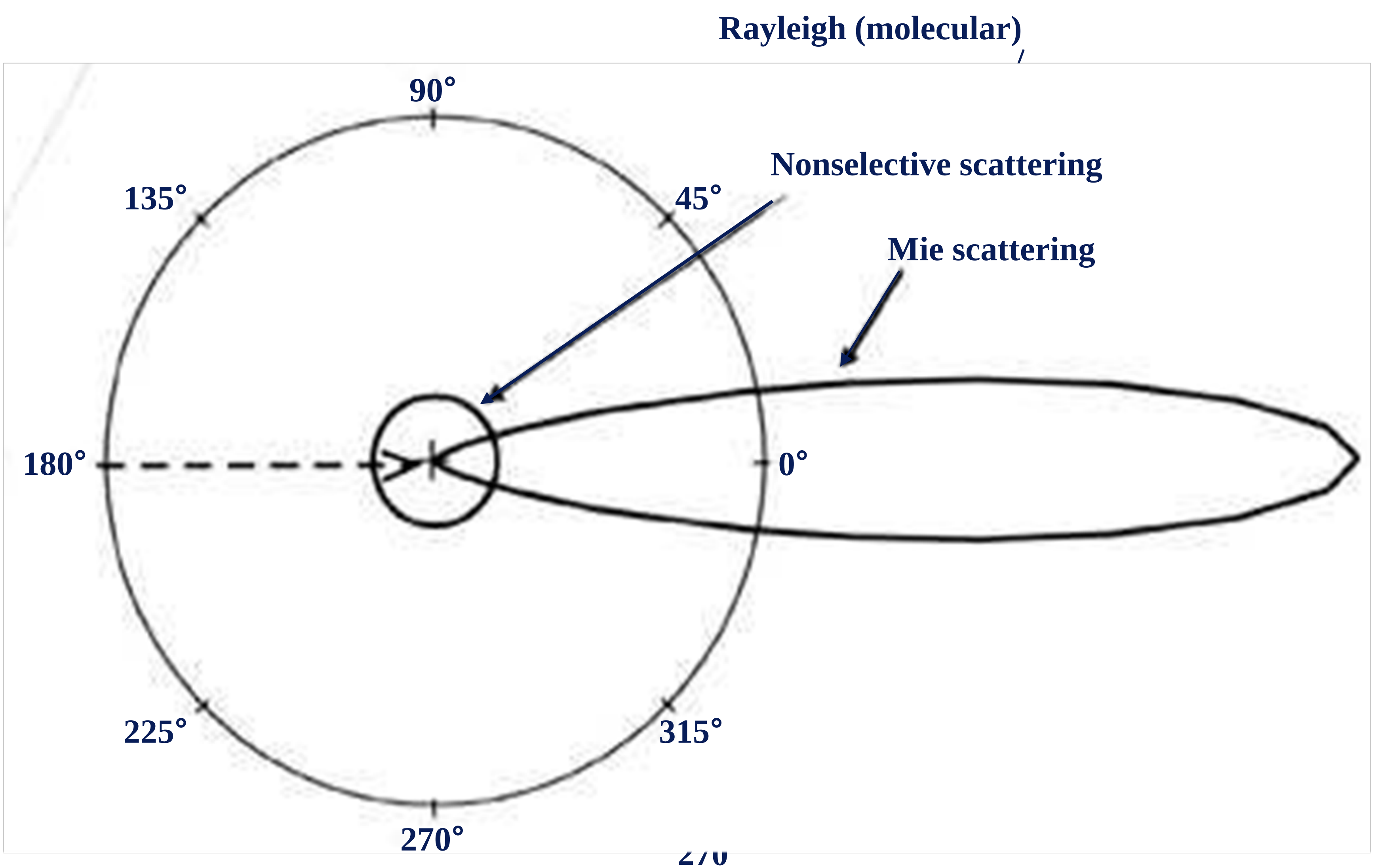
Non-selective Scattering

- Non-selective scattering: when particles in the atmosphere are several times (>10) greater than the wavelength of the radiation
  - All wavelengths of light are scattered, not just blue, green, or red
    - Thus, water droplets scatter all wavelengths of visible light equally well, causing the cloud to appear white (a mixture of all colors of light in approximately equal quantities produces white)
  - Scattering can severely reduce the information content of remotely sensed data to make it difficult to differentiate one object from another

Shapes of Scattering Phase Function (P)



Shapes of Scattering Phase Function (P)



Stokes Vector and Mueller Matrix

The electric field can be resolved into components. E_l and E_r are complex oscillatory functions.

$$\mathbf{E} = E_l\,\mathbf{l} + E_r\,\mathbf{r}$$

The four component Stokes vector can now be defined.

$$I = E_l E_l^* + E_r E_r^* = I_l + I_r$$

$$Q = E_l E_l^* - E_r E_r^* = I_l - I_r$$

$$U = E_l E_r^* + E_r E_l^*$$

$$V = i\,(E_l E_r^* - E_r E_l^*)$$

They are all real numbers and satisfy the relation

$$I^2 = Q^2 + U^2 + V^2$$

The Mueller matrix relates the incident and scattered Stokes vectors

$$\begin{pmatrix} I^s \\ Q^s \\ U^s \\ V^s \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} I^i \\ Q^i \\ U^i \\ V^i \end{pmatrix}$$

Polarization Parameters

I is the radiance (this is what the human eye sees)

Q is the amount of radiation that is polarized in the $0/90^0$ orientation

U is the amount of radiation polarized in the $+/-45^0$ orientation

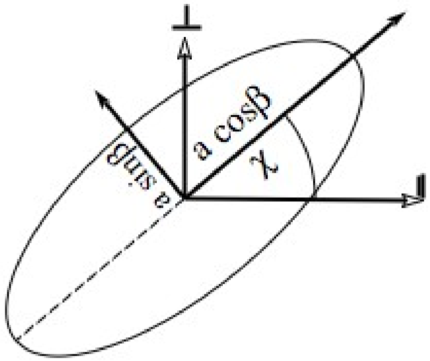
V is the amount of radiation that is right or left circularly polarized

DOP= Degree of polarization= $\sqrt{Q^2 + U^2 + V^2} / I$

DOLP = Degree of linear polarization = $\sqrt{Q^2 + U^2} / I$

DOCP = Degree of circular polarization = $|V|/I$

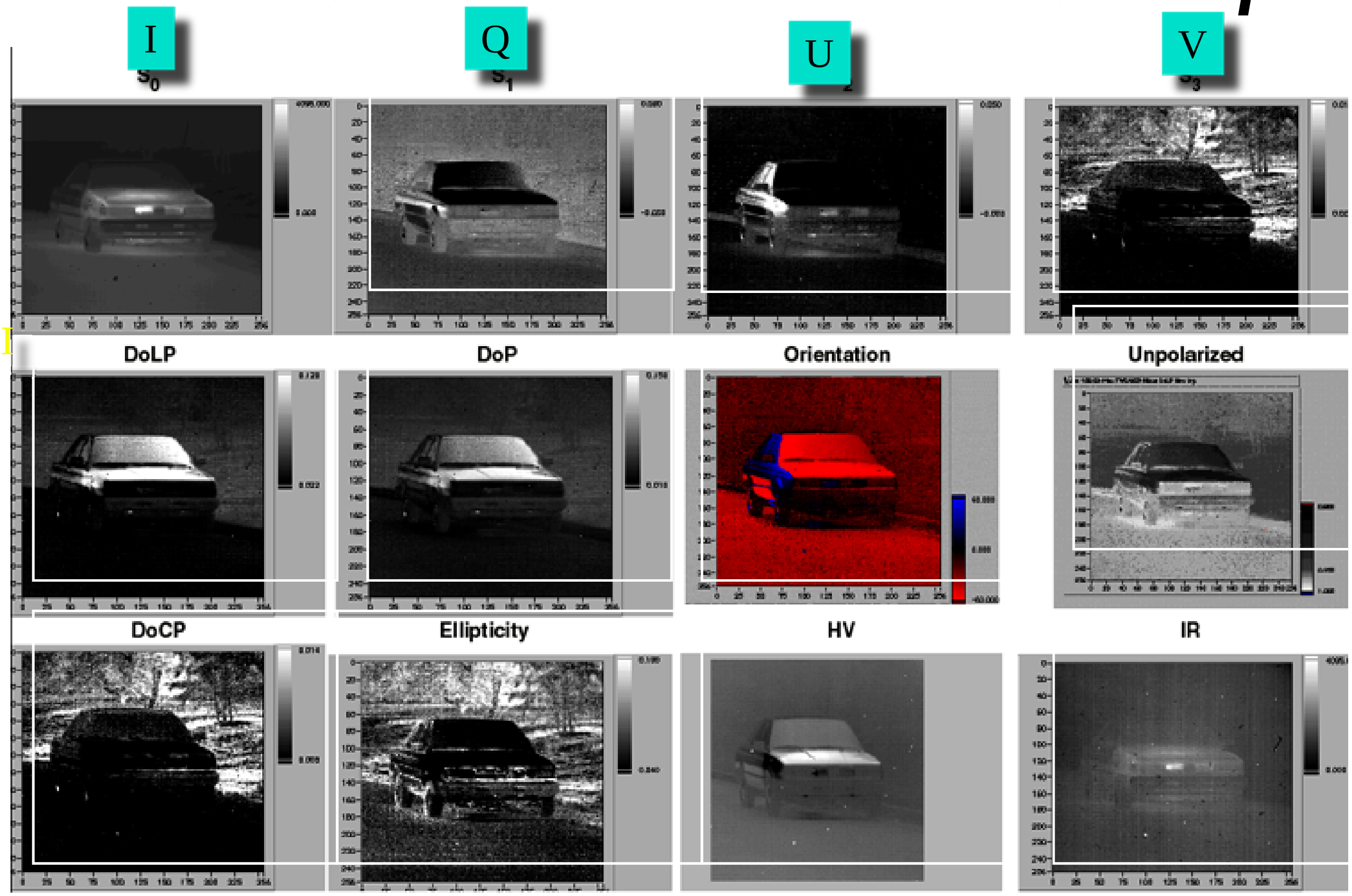
Orientation of plane of polarization = $\chi = \tan^{-1}(U/Q)/2$



Ellipticity= Ratio of semiminor to semimajor axis of polarization ellipse=b/a

$=\tan[(\sin^{-1}(V/I))/2]$

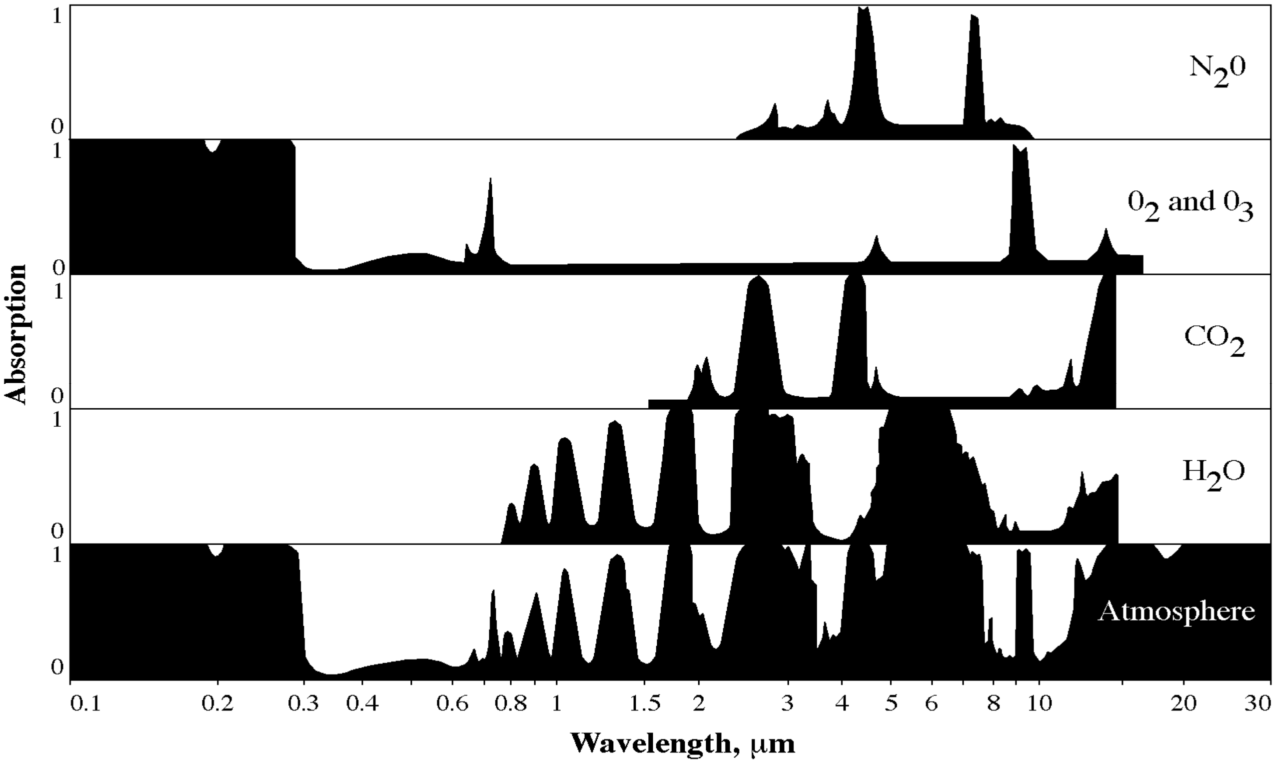
Nissan car viewed in mid-wave infrared



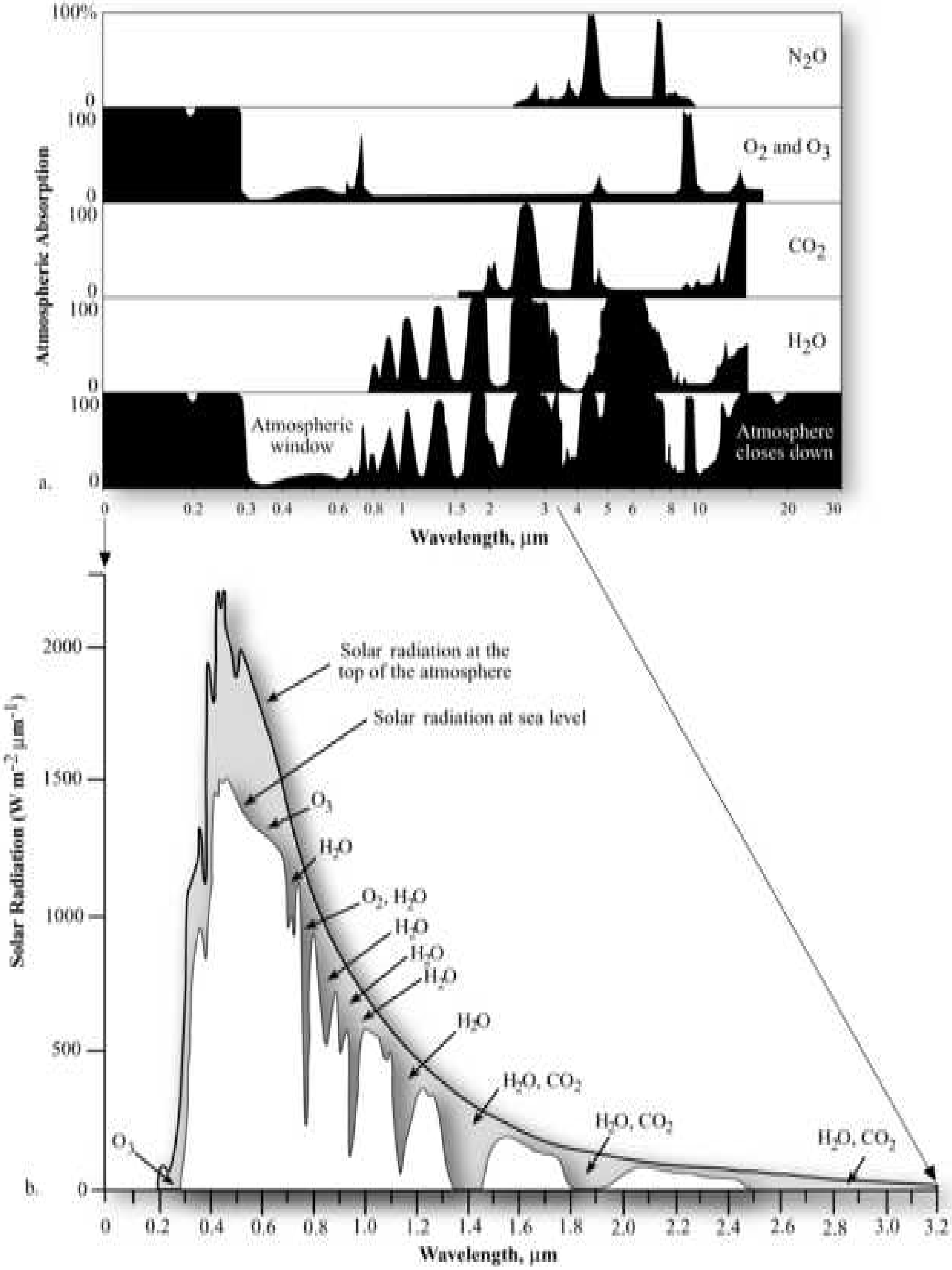
This data was collected using an Amber MWIR InSb imaging array 256x256. The polarization optics consisted of a rotating quarter wave plate and a linear polarizer. Images were taken at eight different positions of the quarter wave plate (22.5 degree increments) over 180 degrees. The data was reduced to the full Stokes vector using a Fourier transform data reduction technique.

4. Atmospheric Absorption

Absorption



- In certain parts of the spectrum such as the visible region (0.4 - 0.7 μm), the atmosphere does not absorb much of the incident energy but transmits it effectively
- Parts of the spectrum that transmit energy effectively are called "atmospheric windows"



- The combined effects of atmospheric absorption, scattering, and reflectance reduce the amount of solar irradiance reaching the Earth's surface at sea level

Rotational and Vibrational Modes of Molecules

Nonlinear molecules: H_2O and O_3 are asymmetric tops with three moments of inertia; gives very complex spectra.

Permanent dipole moment needed for pure rotational lines:

Species	Pure rotation lines
H_2O,O_3	microwave and far IR
O_2	microwave (weak, from permanent magnetic dipole)
CO_2,N_2	none

Three vibrational modes of molecules:

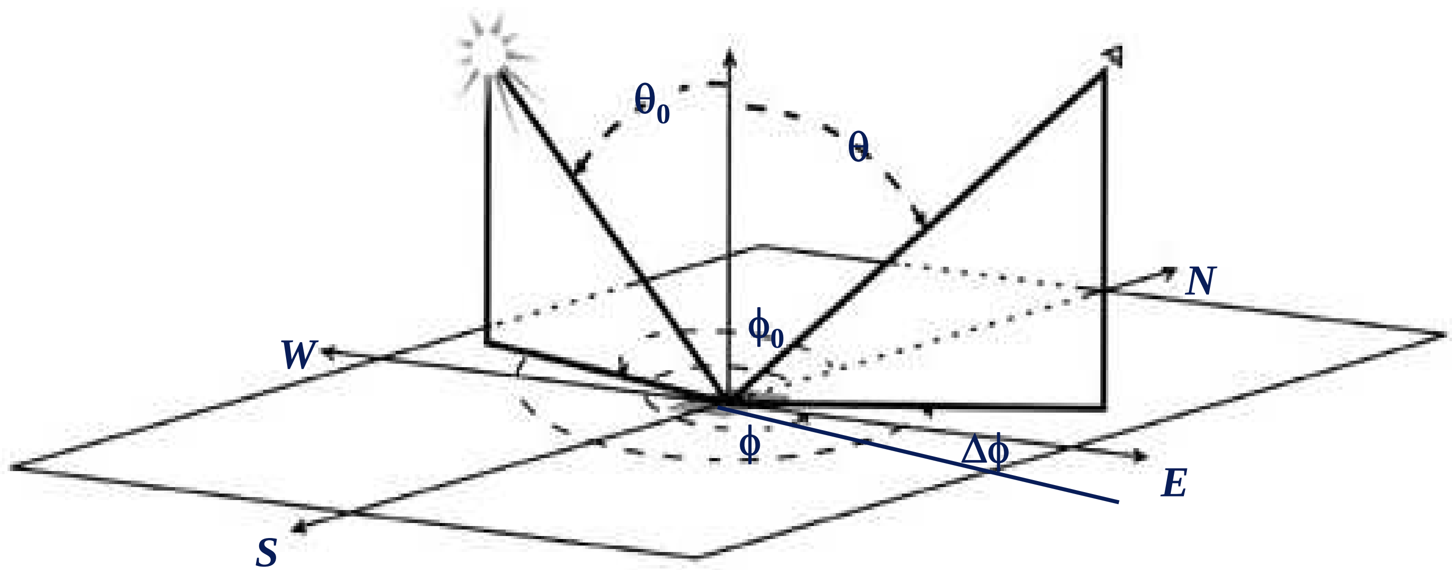
- ν_1 symmetric stretch (2.7 μm H_2O)
- ν_2 bending mode (15.0 μm CO_2, 6.3 μm H_2O)
- ν_3 asymmetric stretch (4.3 μm CO_2, 2.7 μm H_2O)

6. Radiometric Quantities and Reflectance

Definition of Solar Zenith, View Zenith, and Relative Azimuth Angle

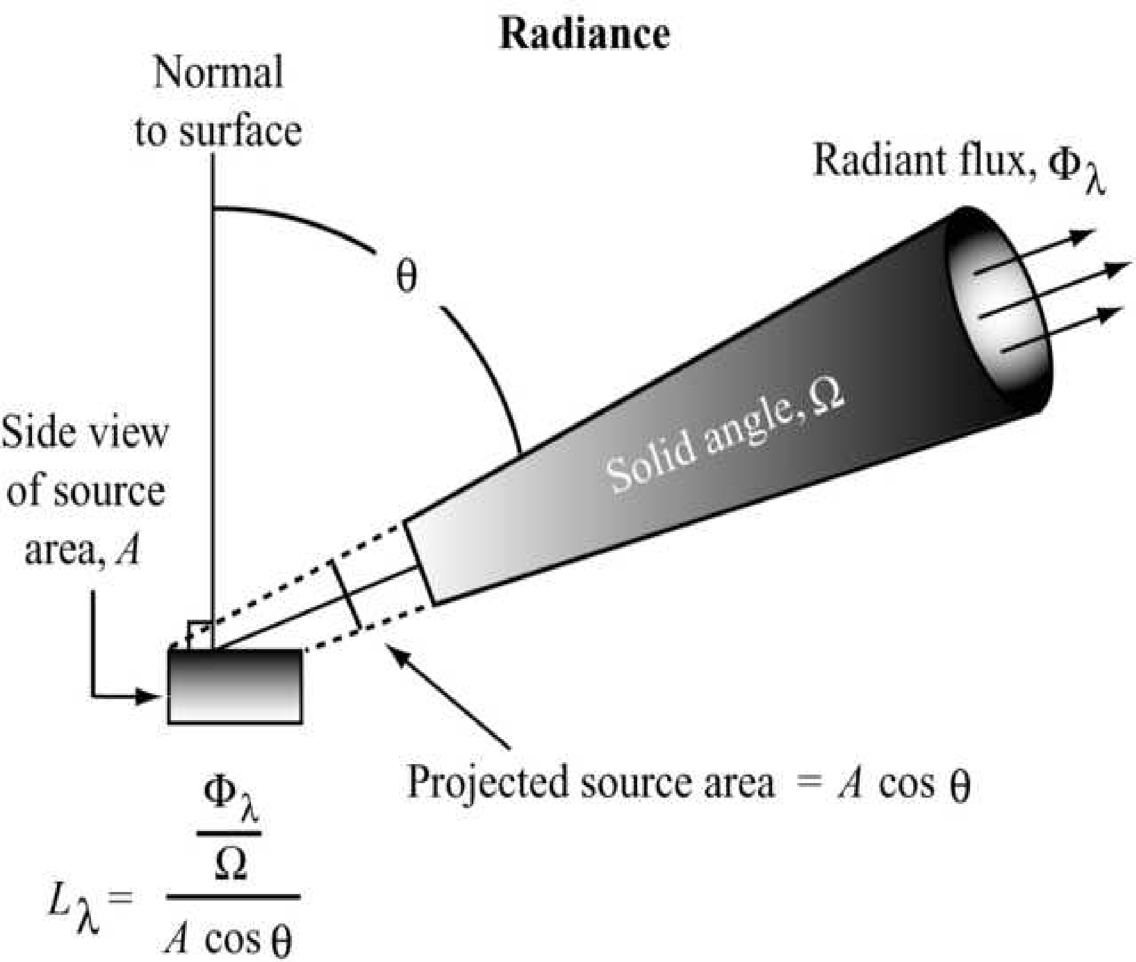
$d\Omega = \sin\theta d\theta d\phi$

z



y

Radiance



The concept of *radiance* (L_λ) leaving a specific projected source area (A) on the ground, in a specific direction (θ), and within a specific solid angle (Ω)

The L_λ is measured in watts per meter squared per steradian ($W\ m^{-2}\ sr^{-1}$)

- We are only interested in the radiant flux in certain wavelengths (Φ_λ) leaving the projected source area (A) within a certain direction (θ) and solid angle (Ω)

Flux (Irradiance) on a Horizontal Surface at the Surface of the Earth

The transmitted flux (irradiance) at the Earth's surface can be calculated as:

$$E(\tau_t, \omega_0; \mu_0, \mu, \phi) = \int_0^{2\pi}\int_0^{\pi} I(\tau_t, \omega_0; \mu, \mu_0, \phi)\mu d\mu d\phi + \mu_0 F_0 \exp(-\tau_t/\mu_0)$$

$$= \mu_0 F_0 \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi} T(\tau_t, \omega_0; \mu, \mu_0, \phi)\mu d\mu d\phi + \exp(-\tau_t/\mu_0)\right]$$

where the transmission function is defined in an analogous manner to reflection function

$$T(\tau_t, \omega_0; \mu, \mu_0, \phi) = \frac{\pi I(\tau_t, \mu, \phi)}{\mu_0 F_0}$$

Definition of Reflection Function

The reflection function is usually a function of at least 5 variables

$$R(\tau_t, \omega_0; \mu, \mu_0, \phi)$$

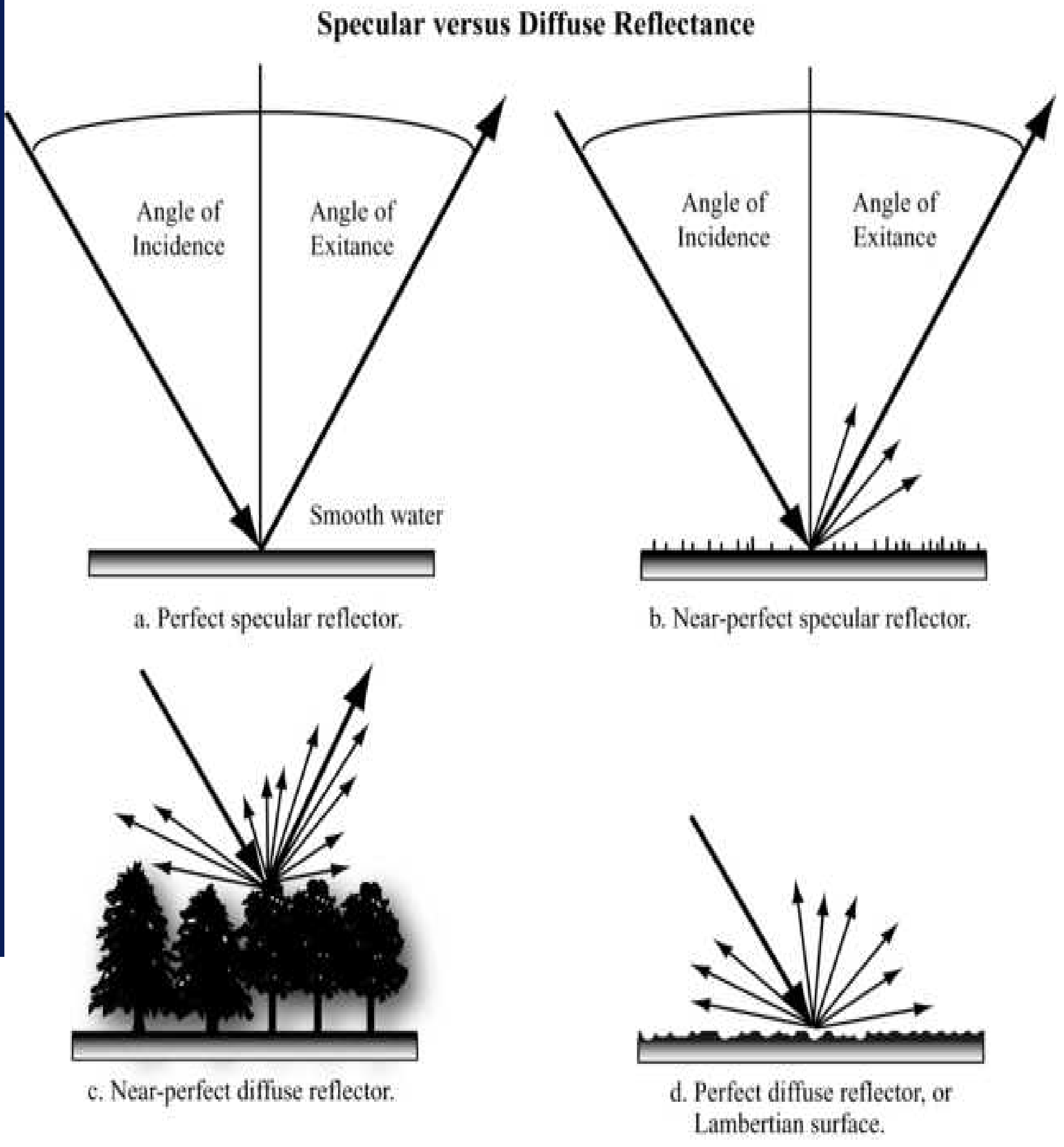
where

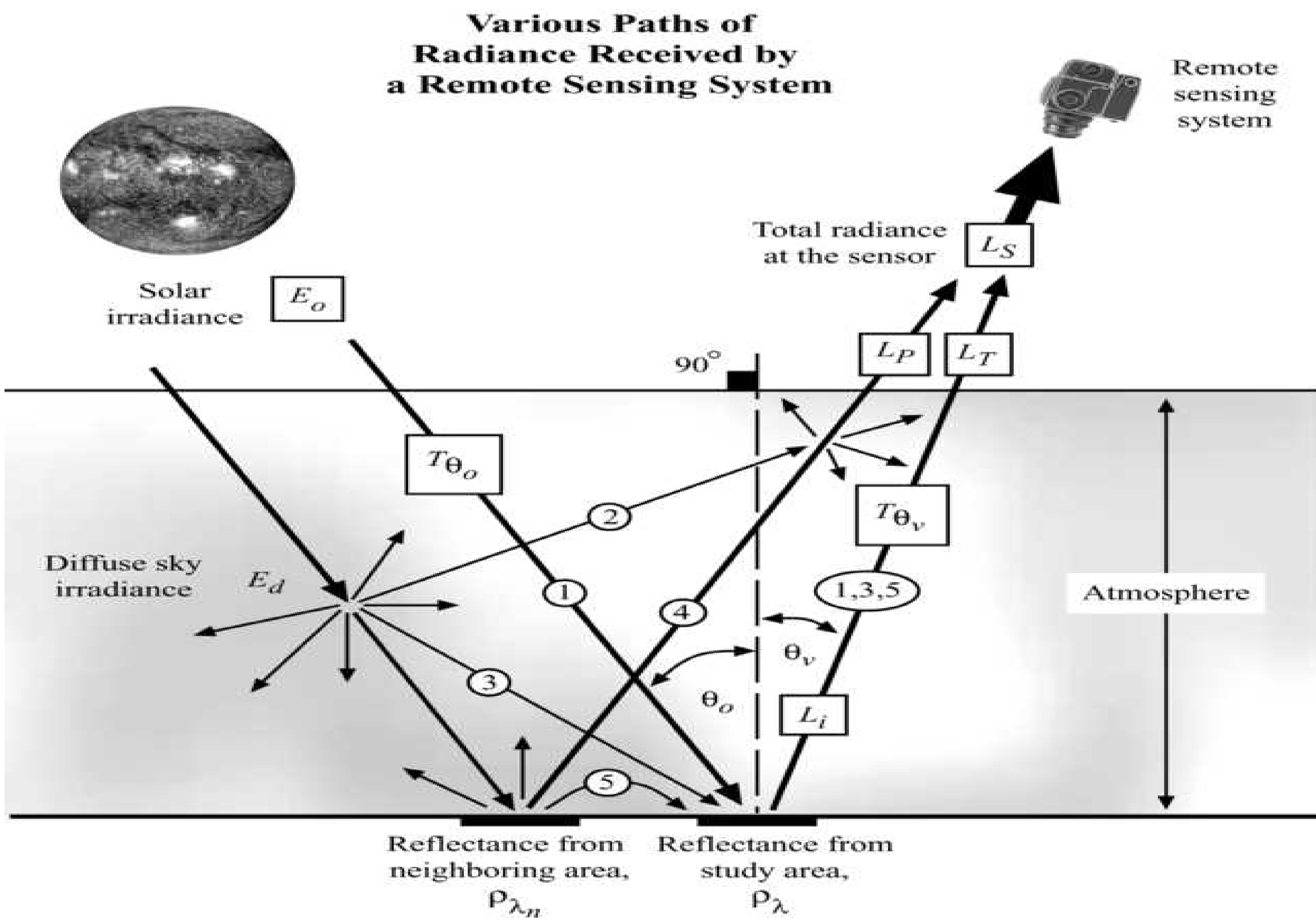
- τ_t = total optical thickness
- ω_0 = the single scattering albedo (ratio of scattering to total extinction)
- μ = absolute value of the cosine of the zenith angle $|\cos\theta|$
- μ_0 = cosine of the solar zenith angle $\cos\theta_0$
- ϕ = relative azimuth angle between the direction of propagation of the emerging radiation and the incident solar direction

Note: R, τ_t, ω_0 are all functions of wavelength λ

Reflectance

- Specular reflection (a): smooth (i.e., the average surface profile is several times smaller than the wavelength of radiation)
- Diffuse reflection (b): rough, the reflected rays go in many directions
- Lambertian surface (d) the radiant flux leaving the surface is constant for any angle of reflectance to the surface normal




Various Paths of
Radiance Received by
a Remote Sensing System
Remote
sensing
system
Total radiance
at the sensor
L_S
Solar
irradiance
E_o
90°
L_P
L_T
T_{θ_o}
T_{θ_v}
2
Diffuse sky
irradiance
E_d
1
4
1,3,5
Atmosphere
θ_v
3
θ_o
L_i
5
Reflectance from
neighboring area,
ρ_{λ_n}
Reflectance from
study area,
ρ_λ

6. Summary

1. **Electromagnetic energy interactions**: interaction with atmosphere, earth, atmosphere, sensor system components (camera, film, emulsion, etc.)
2. **Electromagnetic radiation models (wave/particle)**: three energy transfer ways (conduction, convection & radiation), two EM models (wave ($c=v.\lambda$), particle ($Q=h.v$), $Q\sim\lambda$), S. B. law (total emitted radiance) & Wien's law (peak-wavelength),
3. **Atmospheric scattering**: Rayleigh ($d<<\lambda$), Mie ($d\sim\lambda$), non-selective ($d>>\lambda$), blue sky phenomenon at noon, yellow/radish phenomenon at sunrise/set
4. **Atmospheric absorption ("atmospheric windows"):** 'close down' regions, 'atmospheric windows'
5. **Radiometric quantities and Reflectance**: 1 =refectance+absorption+transmittance, three type reflectances (specular, diffuse & lambertian), Radiance and Irradiance,