Ensemble Kalman Filter: Comparisons with 3D- and 4D-Var and EnKF diagnostics

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Ensemble Kalman Filter: status and new ideas

- EnKF and 4D-Var are in a friendly competition:
- Jeff Whitaker results: EnKF better than GSI (3D-Var)
- Canada (Buehner): 4D-Var & EnKF the same in the NH and EnKF is better in the SH. Hybrid best.
- JMA (Miyoshi): at JMA, EnKF faster than 4D-Var, better in tropics and NH, worse in SH due to model bias.
- EnKF needs no adjoint model, priors, it adapts to changes in obs, it can even estimate ob errors.
- We "plagiarized" ideas and methods developed for 4D-Var and adapted them to the LETKF (Hunt et al., 2007)

Intercomparison of variational, EnKF, and ensemble-4D-Var data assimilation approaches in the context of deterministic NWP

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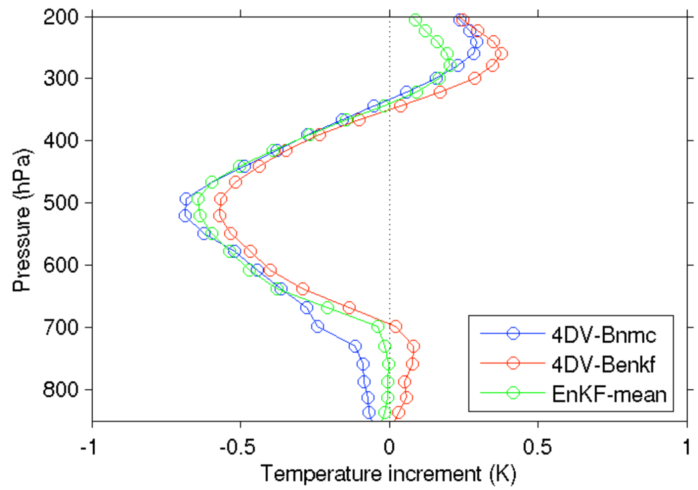
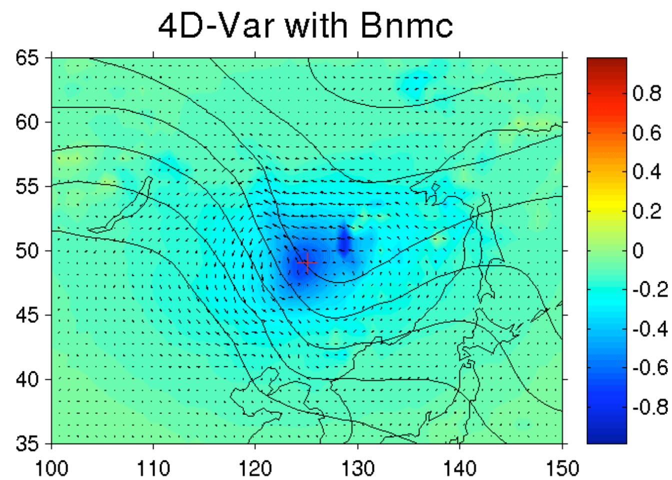
From Seminar at NCEP/EMC
June 9, 2009
See also "4D-Var vs. EnKF" Workshop in Buenos Aires
Google "4D-Var EnKF"

Single observation experiments

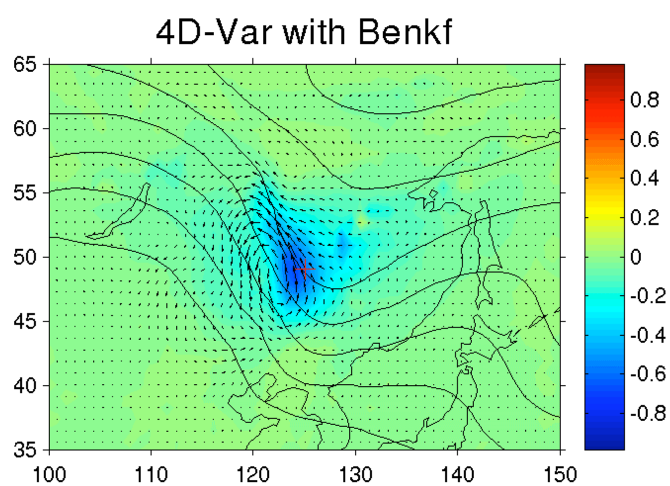
Difference in temporal covariance evolution

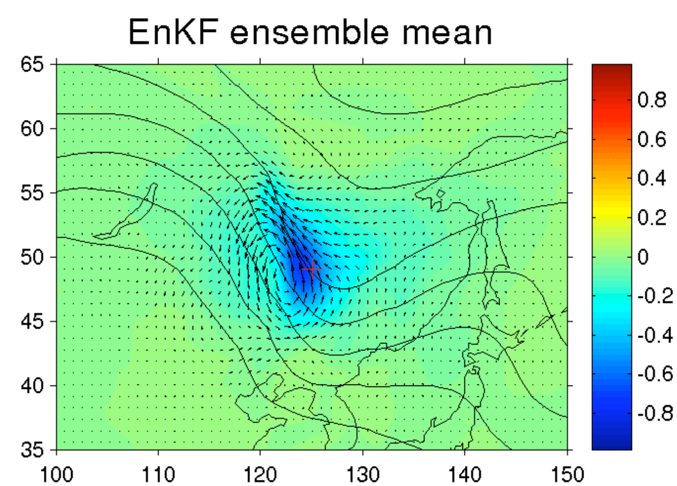
- radiosonde temperature observation at 500hPa
- observation at **middle of assimilation window (+0h)**
- with same B, increments very similar from **4D-Var**, **EnKF**
- contours are 500hPa GZ background state at 0h (ci=10m)





contour plots at 500 hPa





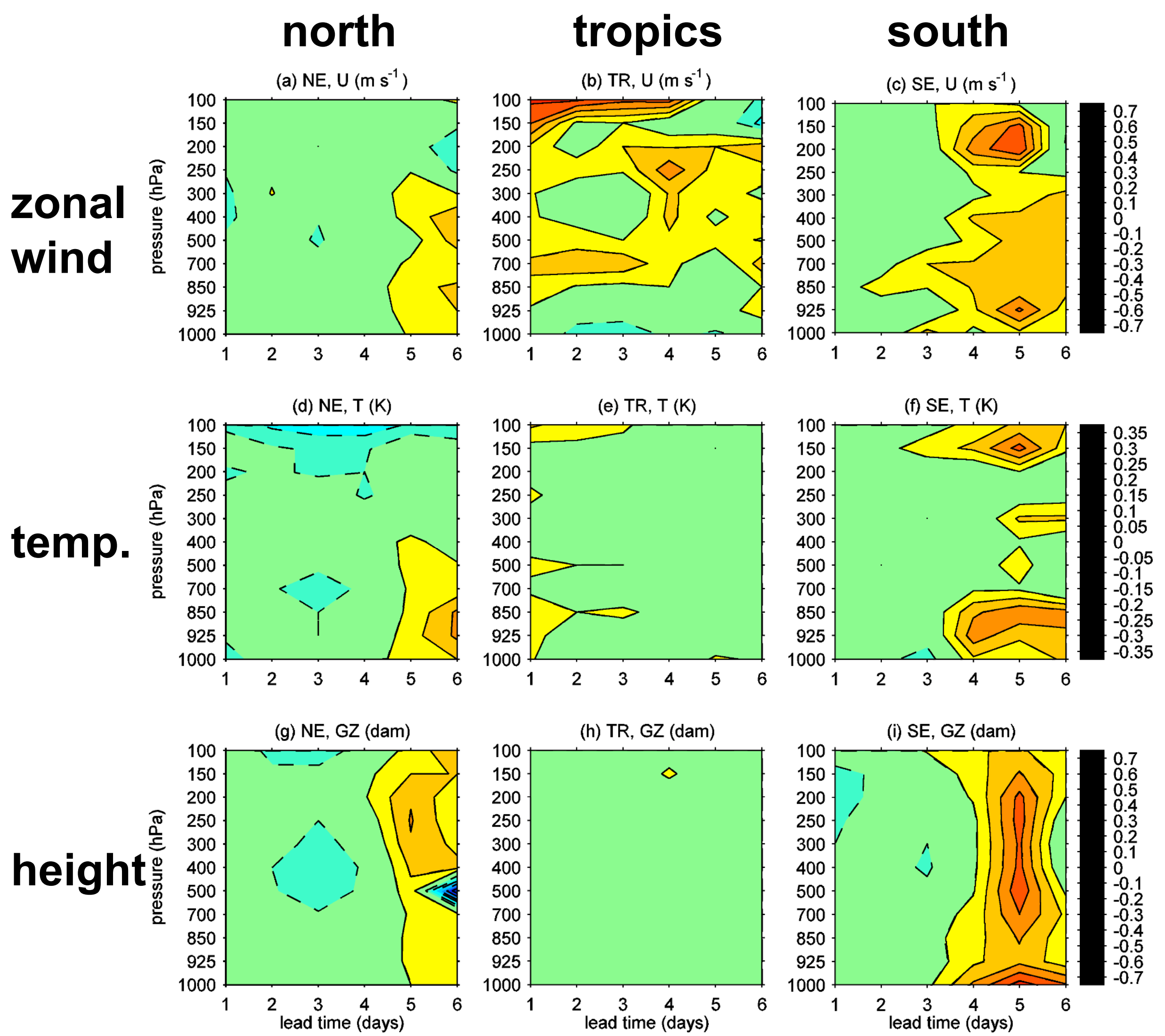
Forecast Results:
EnKF (ens mean) vs. 4D-Var-Bnmc

<u>Difference</u> in stddev relative to radiosondes:

Positive →
EnKF better

Negative →
4D-Var-Bnmc better

EnKF comparable to operational 4D-Var



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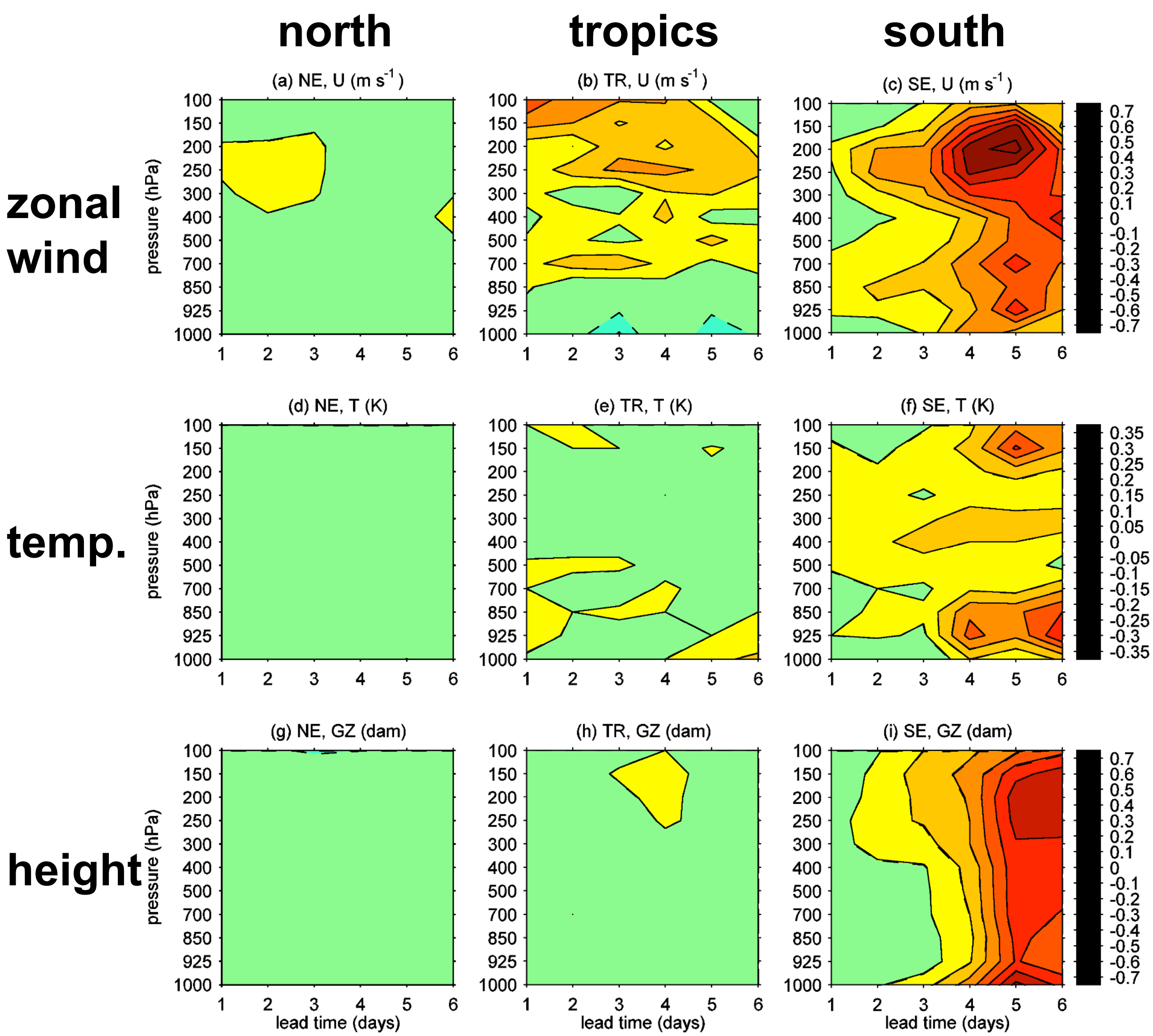
Forecast Results (Hybrid): 4D-Var-Benkf vs. 4D-Var-Bnmc

Difference in stddev relative to radiosondes:

Positive → 4D-Var-Benkf better

Negative → 4D-Var-Bnmc better

Hybrid better than operational 4D-Var



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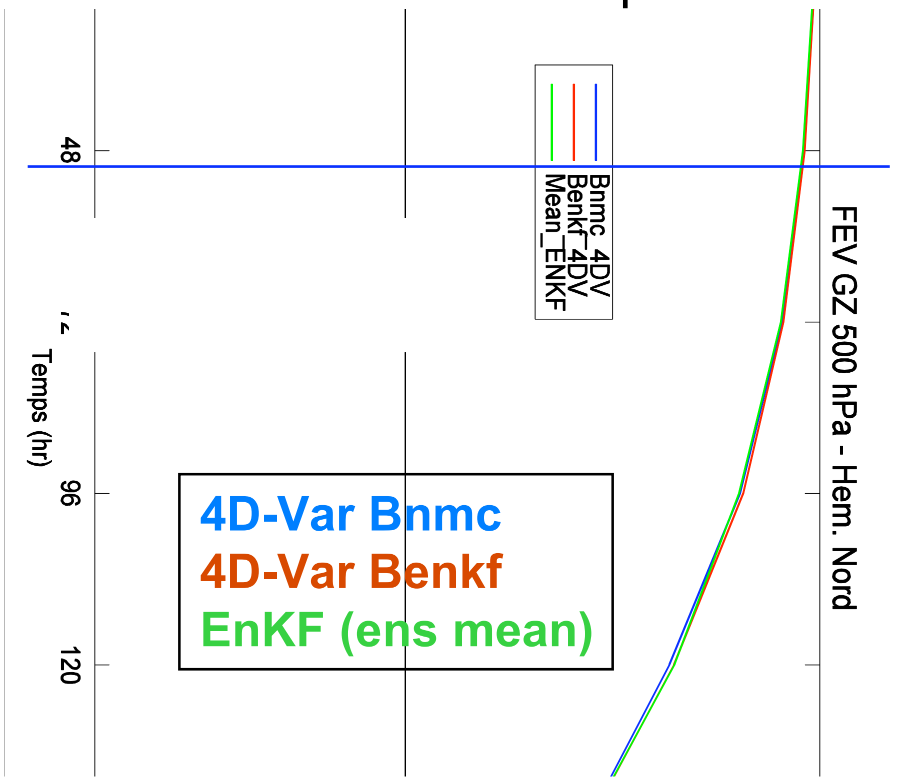
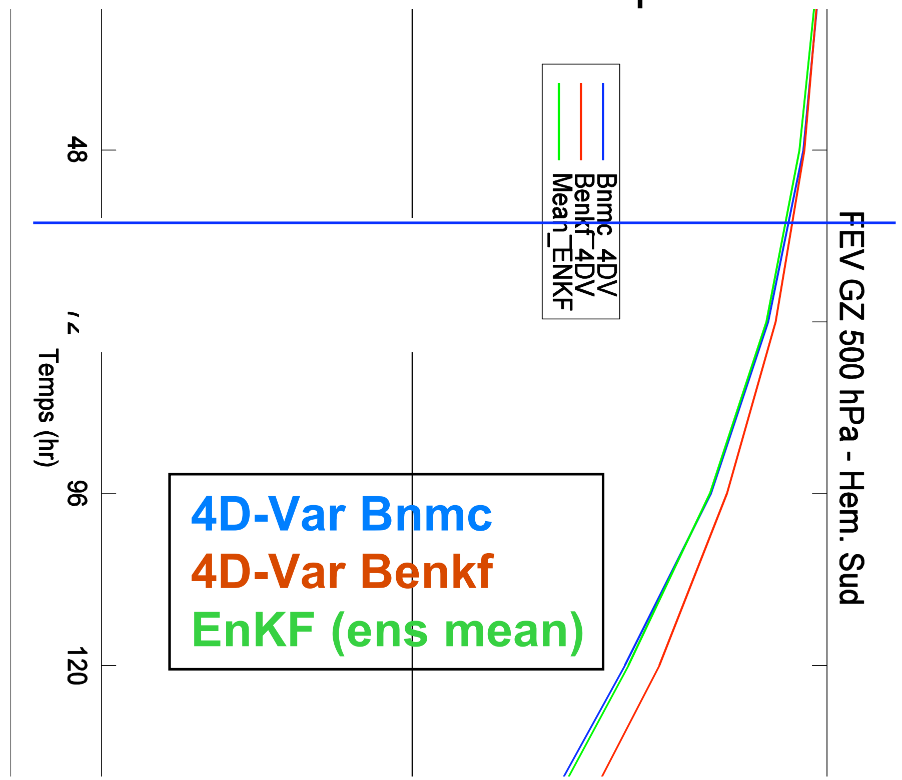
Results – 500hPa GZ anomaly correlation

Verifying analyses from 4D-Var with Bnmc

Northern extra-tropics



Southern extra-tropics



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SORRY THAT THIS FIGURE MAY
APPEAR ROTATED BY MICROSOFT!!!

Forecast Results – Precipitation

24-hour accumulation verified against GPCP analyses

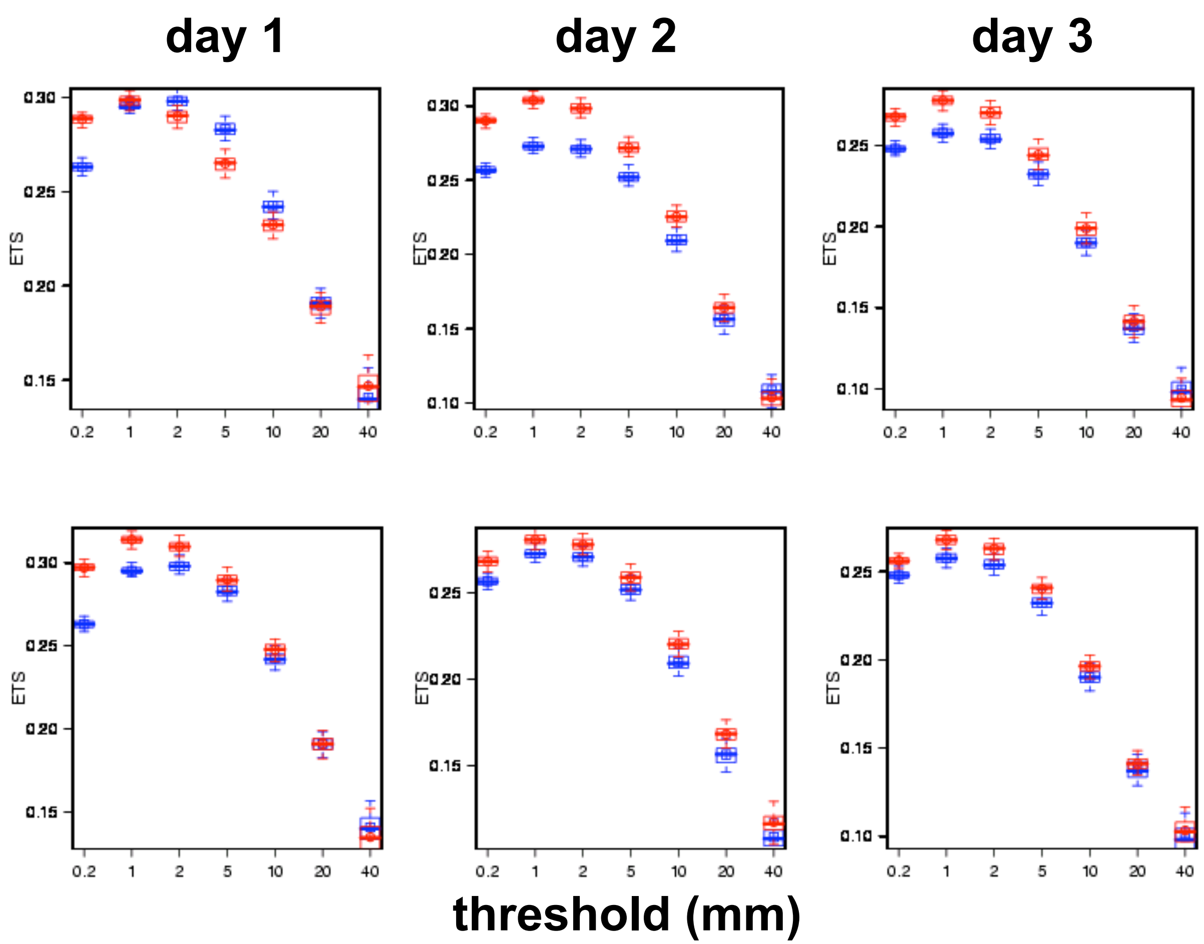
Equitable Threat Score for Tropics

EnKF (ens mean)
4D-Var-Bnmc

EnKF better than operational 4D-Var

4D-Var-Benkf
4D-Var-Bnmc

Hybrid better than operational 4D-Var



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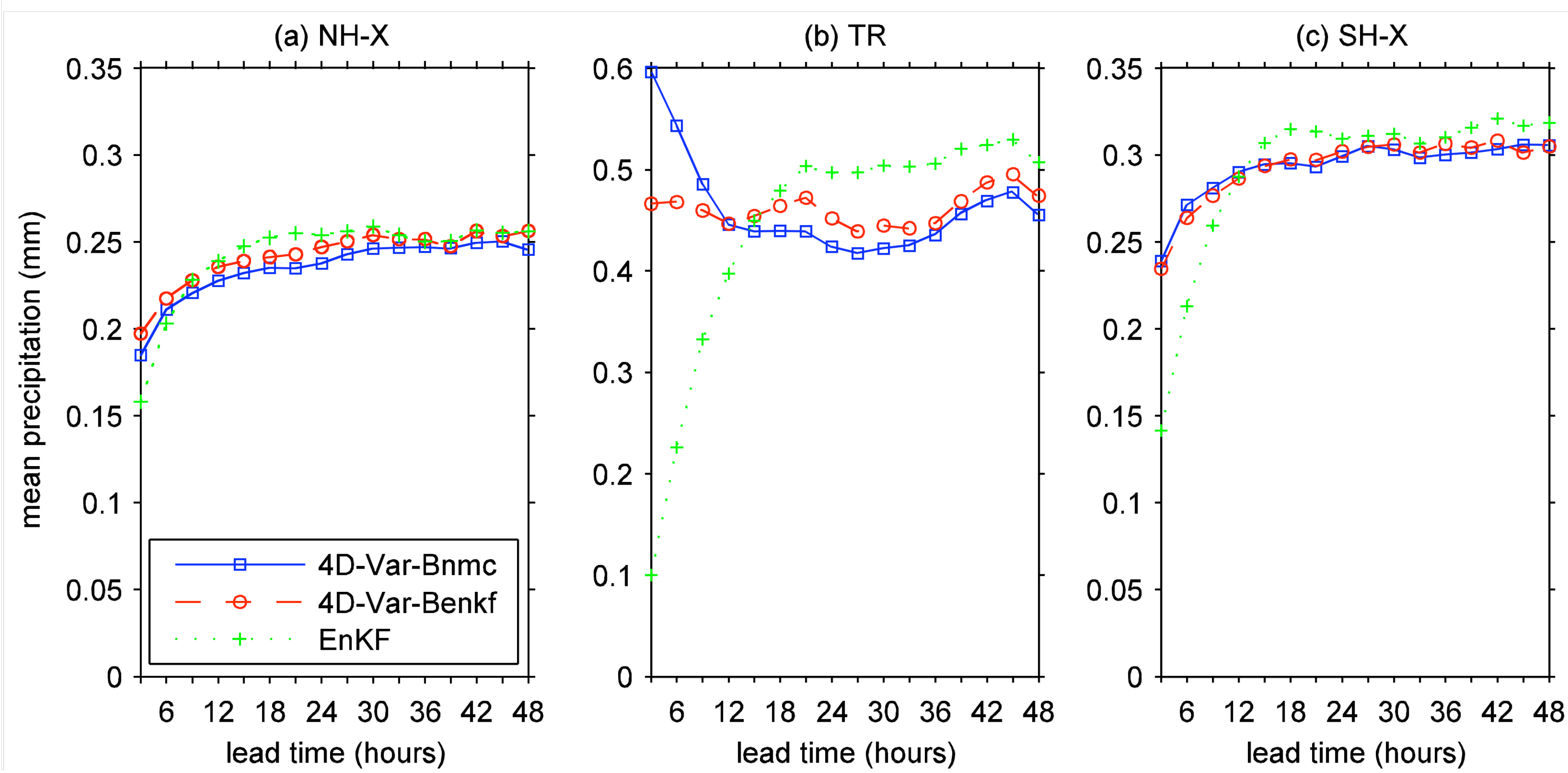
Forecast Results – Precipitation

Evolution of mean 3-hour accumulated precipitation



EnKF has tropical spinup

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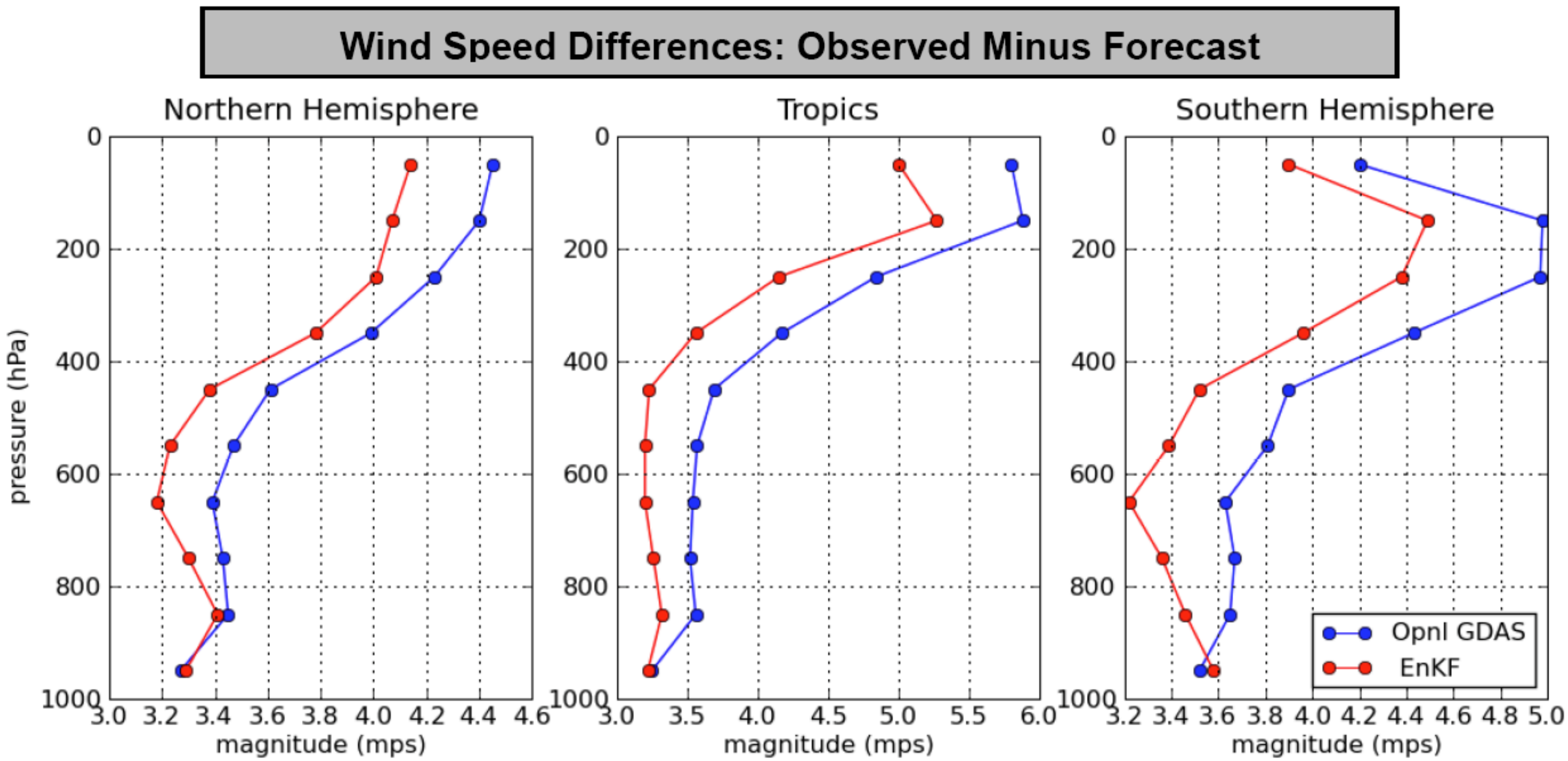
Whitaker: Comparison of T190, 64 members EnKF with T382 operational GSI, same observations (JCSDA, 2009)



Vertical profiles of the RMS difference between six hour forecasts and in-situ observations for the period 2007120700 – 2008010718. Observations are aggregated in 100 hPa layers. The red curve is for the ensemble mean of the experimental 64-member T190 EnKF system, and the blue curve is for the T382 GSI-based GDAS system operational in December 2007.

Conclusions from a clean comparison of GSI and EnKF (Whitaker, 2009)

- ✓ At T190/L64 resolution - 64 members - EnKF is better than the operational GSI (at T382/L64)
- ✓ It is computationally competitive with 3D-Var if the forecast costs are covered by the operational ensemble requirement

Conclusions from a clean comparison of JMA 4D-Var and LETKF (Miyoshi et al. 08)

- ✓ At the same resolution LETKF is <u>faster</u> than the operational 4D-Var, <u>better</u> in the tropics and NH, <u>worse</u> in SH due to a model bias
- ✓ Plan to test simple low-dim method to correct model bias

There are several types of EnKF

1. Perturbed obs (e.g., Houtekamer and Mitchell)
2. Square root filters (e.g., Whitaker and Hamill)

Most filters get their speed from assimilating one observation at a time

The LETKF (Hunt 2005) assimilates all obs simultaneously and get its speed from local processing of each grid point

Because it is a Transform Square root filter, the LETKF analysis ensemble is explicitly expressed as a linear combination of the forecast ensemble

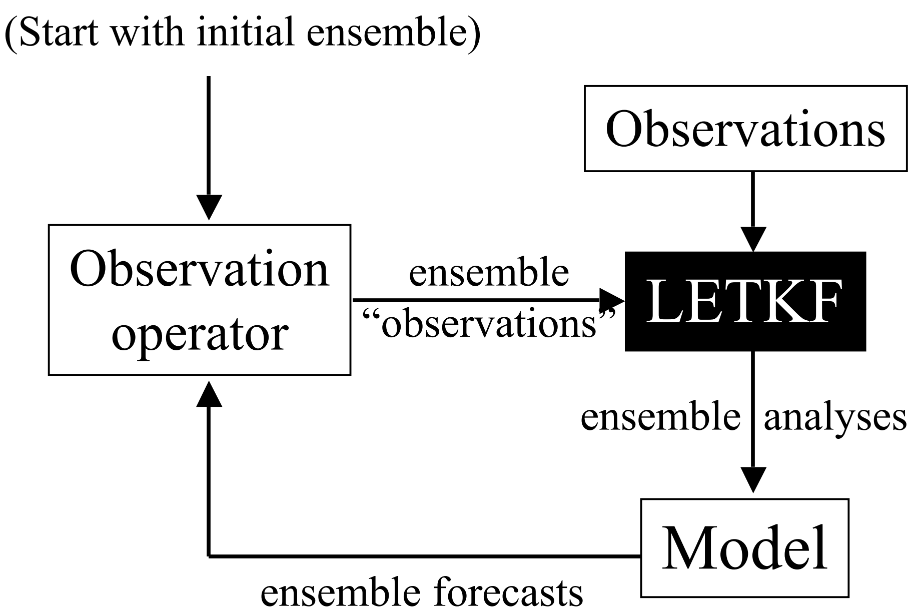
This has a number of nice properties, so here we will focus on the LETKF

Diagnostic tools that improve LETKF/EnKF

We adapted ideas that were inspired by 4D-Var:

- ✓ **No-cost smoother** (Kalnay et al, Tellus 2007)
- ✓ "**Outer loop**", nonlinearities and long windows (Yang and Kalnay)
- ✓ Accelerating the **spin-up** (Kalnay and Yang, 2008)
- ✓ **Forecast sensitivity** to observations (Liu and Kalnay, QJ, 2008)
- ✓ **Analysis sensitivity** to observations and **cross-validation** (Liu et al., QJ, 2009)
- ✓ **Coarse** analysis resolution without degradation (Yang et al., QJ, 2009)
- ✓ Low-dimensional **model bias correction** (Li et al., MWR, 2009)
- ✓ Simultaneous estimation of **optimal inflation** and **observation errors** (Li et al., QJ, 2009).

Local Ensemble Transform Kalman Filter (Ott et al, 2004, Hunt et al, 2004, 2007)

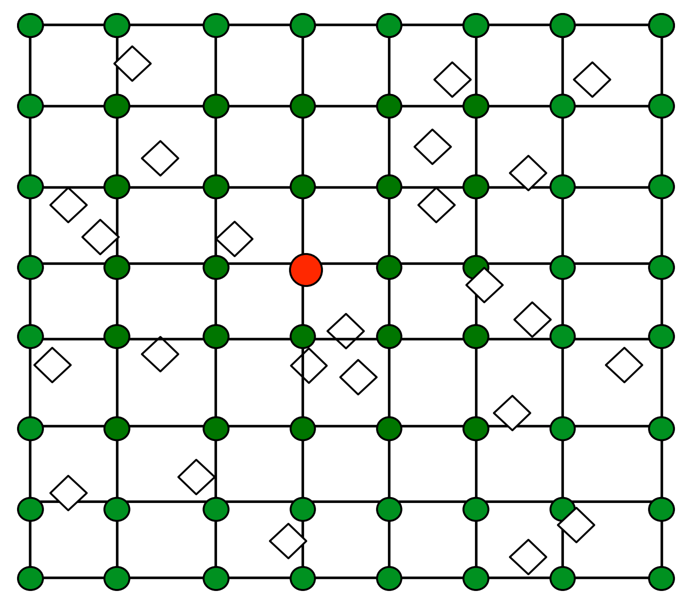


- Model independent (black box)
- Obs. assimilated simultaneously at each grid point
- 100% parallel: fast
- No **adjoint** needed
- **4D LETKF extension**

Localization based on observations

Perform data assimilation in a local volume, choosing observations

The state estimate is updated at the central grid red dot

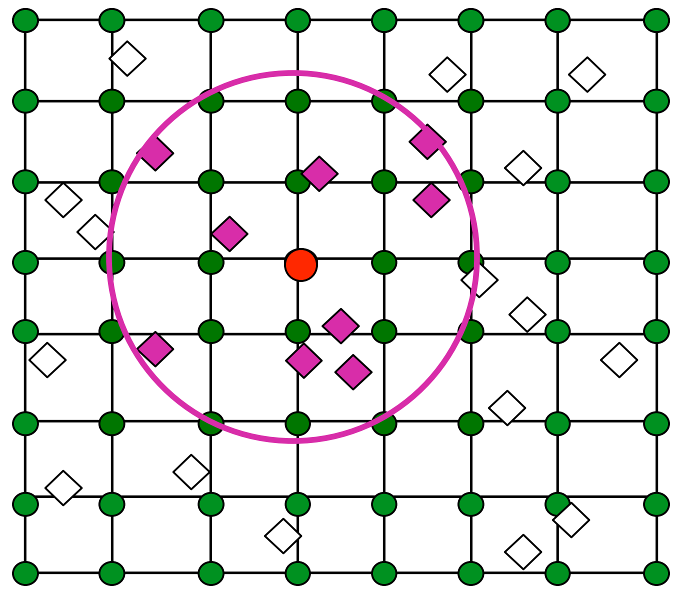


Localization based on observations

Perform data assimilation in a local volume, choosing observations

The state estimate is updated at the central grid red dot

All observations (purple diamonds) within the local region are assimilated



The LETKF algorithm can be described **in a single slide**!

Local Ensemble Transform Kalman Filter (LETKF)

Globally:

Forecast step: $\mathbf{x}^b_{n,k} = M_n\left(\mathbf{x}^a_{n-1,k}\right)$

Analysis step: construct $\mathbf{X}^b = \left[\mathbf{x}^b_1 - \overline{\mathbf{x}}^b \,|\, \ldots \,|\, \mathbf{x}^b_K - \overline{\mathbf{x}}^b\right];$

$$\mathbf{y}^b_i = H(\mathbf{x}^b_i);\ \mathbf{Y}^b_n = \left[\mathbf{y}^b_1 - \overline{\mathbf{y}}^b \,|\, \ldots \,|\, \mathbf{y}^b_K - \overline{\mathbf{y}}^b\right]$$

Locally: Choose for each grid point the observations to be used, and compute the local analysis error covariance and perturbations in ensemble space:

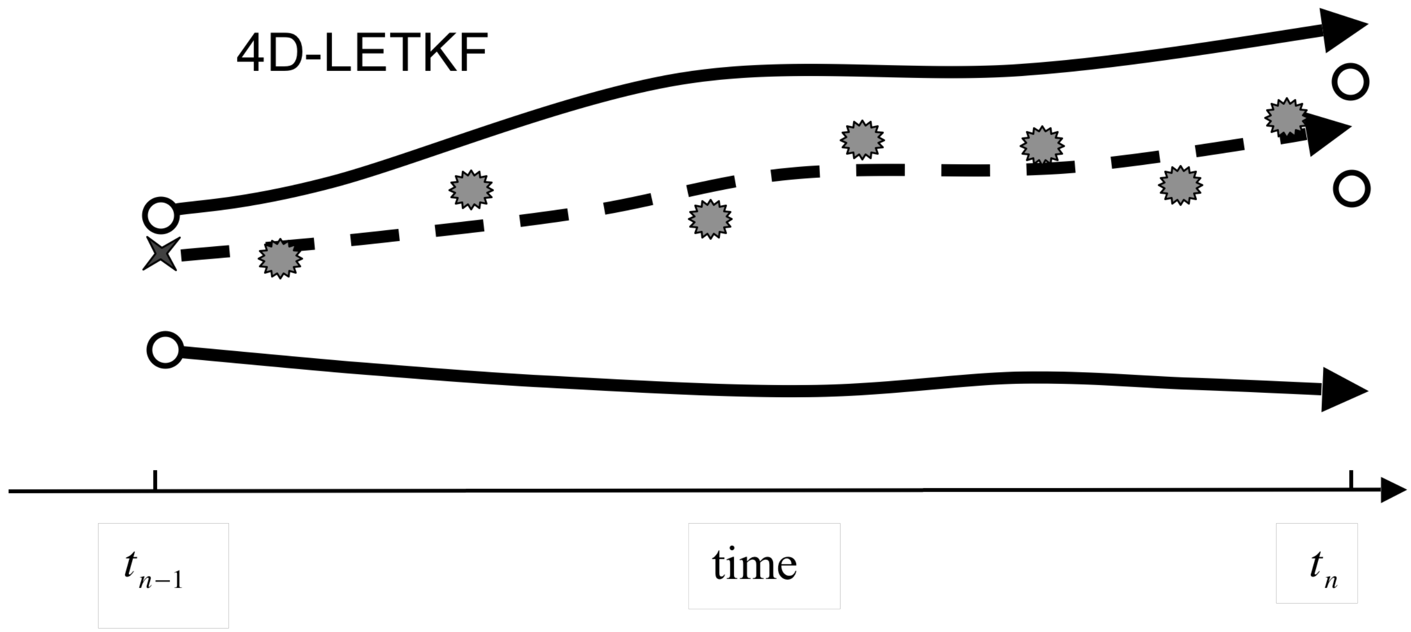
$$\tilde{\mathbf{P}}^a = \left[(K-1)\mathbf{I} + \mathbf{Y}^{bT}\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1};\ \mathbf{W}^a = [(K-1)\tilde{\mathbf{P}}^a]^{1/2}$$

Analysis mean in ensemble space: $\overline{\mathbf{w}}^a = \tilde{\mathbf{P}}^a\mathbf{Y}^{bT}\mathbf{R}^{-1}(\mathbf{y}^o - \overline{\mathbf{y}}^b)$

and add to $\mathbf{W}^a$ to get the analysis ensemble in ensemble space.

The new ensemble analyses in model space are the columns of $\mathbf{X}^a_n = \mathbf{X}^b_n\mathbf{W}^a + \overline{\mathbf{x}}^b$. Gathering the grid point analyses forms the new global analyses. Note that the the output of the LETKF are analysis weights $\overline{\mathbf{w}}^a$ and perturbation analysis matrices of weights $\mathbf{W}^a$. **These weights multiply the ensemble forecasts.**

No-cost LETKF smoother (✕): apply at t_{n-1} the same weights found optimal at t_n. It works for 3D- or 4D-LETKF



The no-cost smoother makes possible:

- Outer loop (like in 4D-Var)
- "Running in place" (faster spin-up)
- Use of future data in reanalysis
- Ability to use longer windows

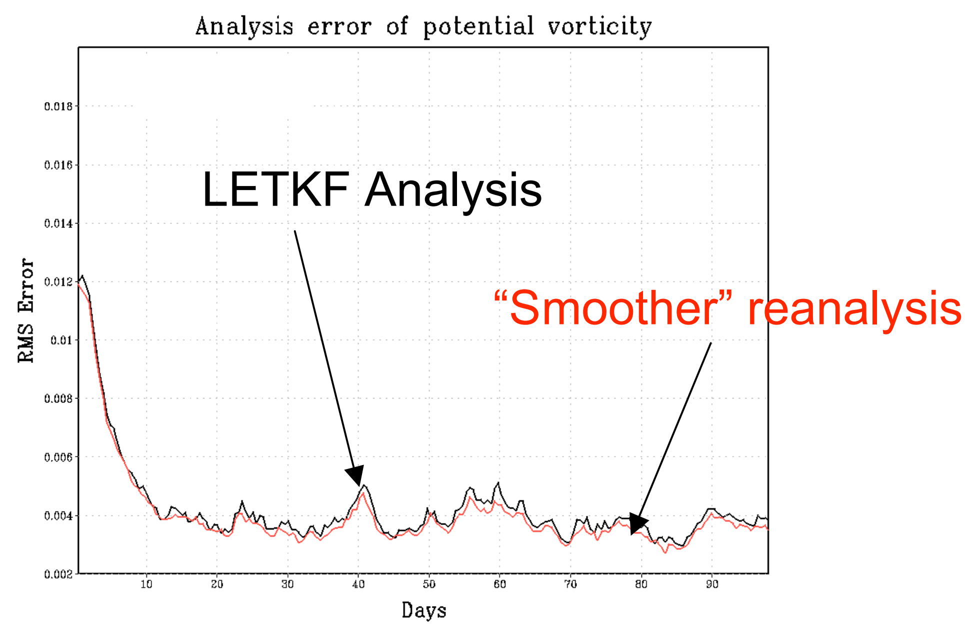
No-cost LETKF smoother tested on a QG model: It works!

LETKF analysis
at time n

$$\overline{\mathbf{x}}_n^a = \overline{\mathbf{x}}_n^f + \mathbf{X}_n^f \overline{\mathbf{w}}_n^a$$

Smoother analysis
at time n-1

$$\tilde{\mathbf{x}}_{n-1}^a = \overline{\mathbf{x}}_{n-1}^f + \mathbf{X}_{n-1}^f \overline{\mathbf{w}}_n^a$$



This very simple smoother allows us to go back and forth in time within an assimilation window:
it allows assimilation of **future** data in reanalysis

Nonlinearities and “outer loop”

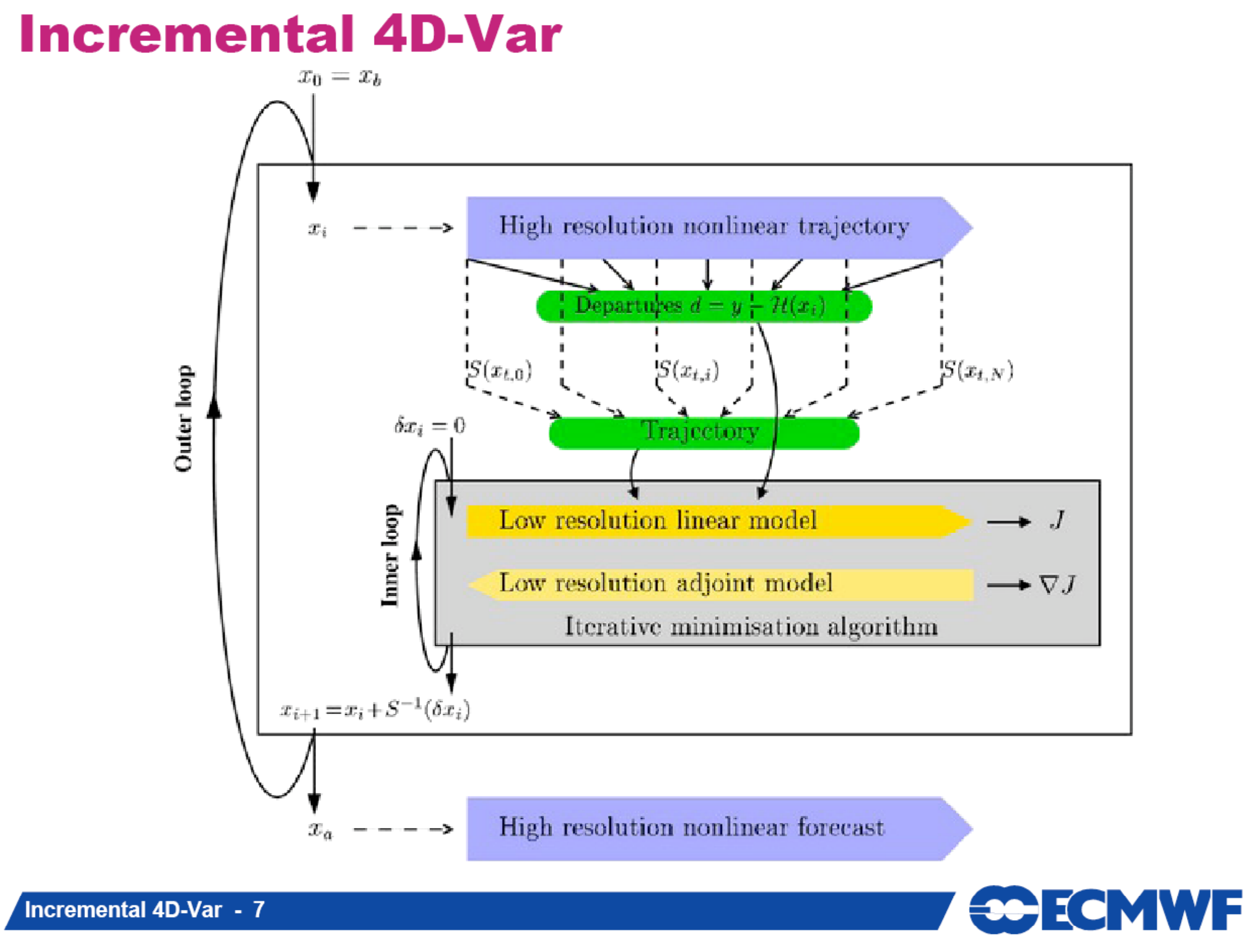
- The main disadvantage of EnKF is that it cannot handle nonlinear (non-Gaussian) perturbations and therefore needs short assimilation windows.
- It doesn’t have the **outer loop** so important in 3D-Var and 4D-Var (DaSilva, pers. comm. 2006)

Lorenz -3 variable model (Kalnay et al. 2007a Tellus), RMS analysis error

	4D-Var	LETKF
Window=8 steps	0.31	**0.30** (linear window)
Window=25 steps	**0.53**	0.66 (nonlinear window)

Long windows + Pires et al. => 4D-Var clearly wins!

“Outer loop” in 4D-Var



Nonlinearities, “Outer Loop” and “Running in Place”

Outer loop: similar to 4D-Var: use the final weights to correct only the mean initial analysis, keeping the initial perturbations. Repeat the analysis once or twice. It centers the ensemble on a more accurate nonlinear solution.

Lorenz -3 variable model RMS analysis error

	4D-Var	LETKF	LETKF +outer loop	LETKF +RIP
Window=8 steps	0.31	0.30	**0.27**	0.27
Window=25 steps	0.53	0.66	**0.48**	**0.39**

“Running in place” smoothes both the analysis and the analysis error covariance and iterates a few times…

"Running in Place": like the outer loop but updating also the covariance

Cold-start the EnKF from any initial ensemble mean and random perturbations at t_0, and integrate the initial ensemble to t_1. The "running in place" loop with $n=1$, is:

a) Perform a standard EnKF analysis and obtain the analysis weights at t_n, saving the mean square observations minus forecast (OMF) computed by the EnKF.

b) Apply the no-cost smoother to obtain the smoothed analysis ensemble at t_{n-1} by using the same weights obtained at t_n.

c) Perturb the smoothed analysis ensemble with a small amount of random Gaussian perturbations, similar to additive inflation.

d) Integrate the perturbed smoothed ensemble to t_n. If the forecast fit to the observations is smaller than in the previous iteration according to some criterion, go to a) and perform another iteration. If not, let $t_{n-1} \leftarrow t_n$ and proceed to the next assimilation window.

Running in Place algorithm (notes)

Notes:

c) *Perturb the smoothed analysis ensemble with a small amount of random Gaussian perturbations, a method similar to additive inflation.*

This perturbation has two purposes:

1) Avoid reaching the same analysis as before, and
2) Encourage the ensemble to explore new unstable directions

d) *Convergence criterion*: if

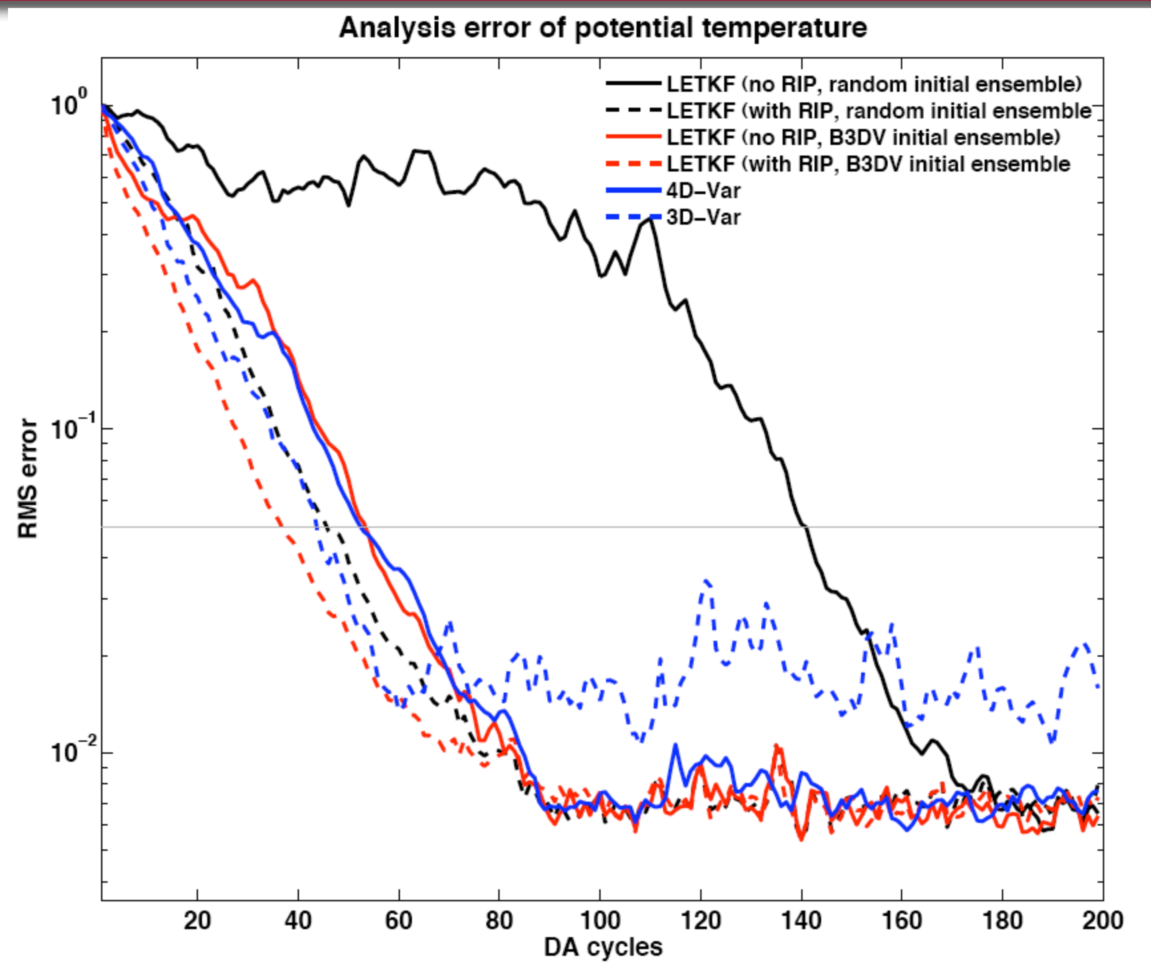
with do another iteration. Otherwise go to the next assimilation window.

and keep the number of iterations low

$$\frac{OMF^2(iter) - OMF^2(iter+1)}{OMF^2(iter)} > \varepsilon$$

$$\varepsilon \sim 5\%$$

Results with a QG model



Spin-up depends on initial perturbations, but RIP works well even with random perturbations. It becomes as fast as 4D-Var (blue). RIP takes only 2-4 iterations.

Results with a QG model

	LETKF Random initial ensemble		LETKF B3DV initial ensemble		LETKF, Random initial ensemble	Variational	
	No RIP	With RIP	No RIP	With RIP	Fixed 10 iterations RIP	3D-Var B3DV	4D-Var 0.05B3DV
Spin-up: DA cycles to reach 5% error	**141**	**46**	**54**	**37**	**37**	**44**	**54**
RMS error ($x10^{-2}$)	0.5	0.54	0.5	0.52	1.16	1.24	0.54

LETKF spin-up from random perturbations: 141 cycles. With RIP: 46 cycles

LETKF spin-up from 3D-Var perts. 54 cycles. With RIP: 37 cycles

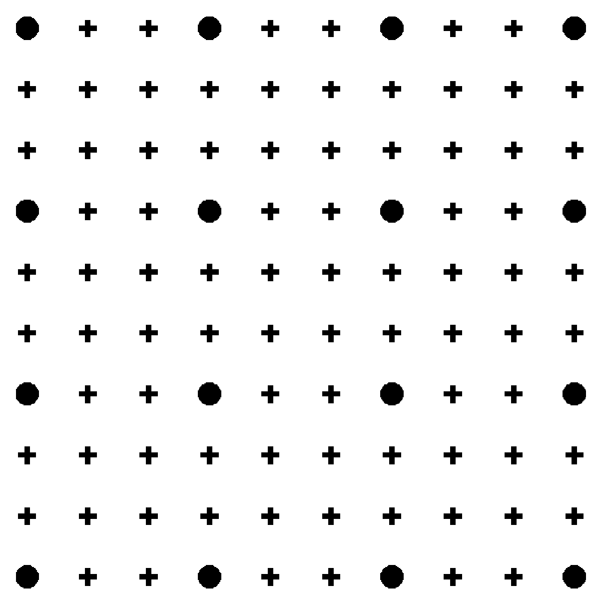
4D-Var spin-up using 3D-Var prior: 54 cycles.

RIP is robust, with or without prior information

Coarse analysis with interpolated weights

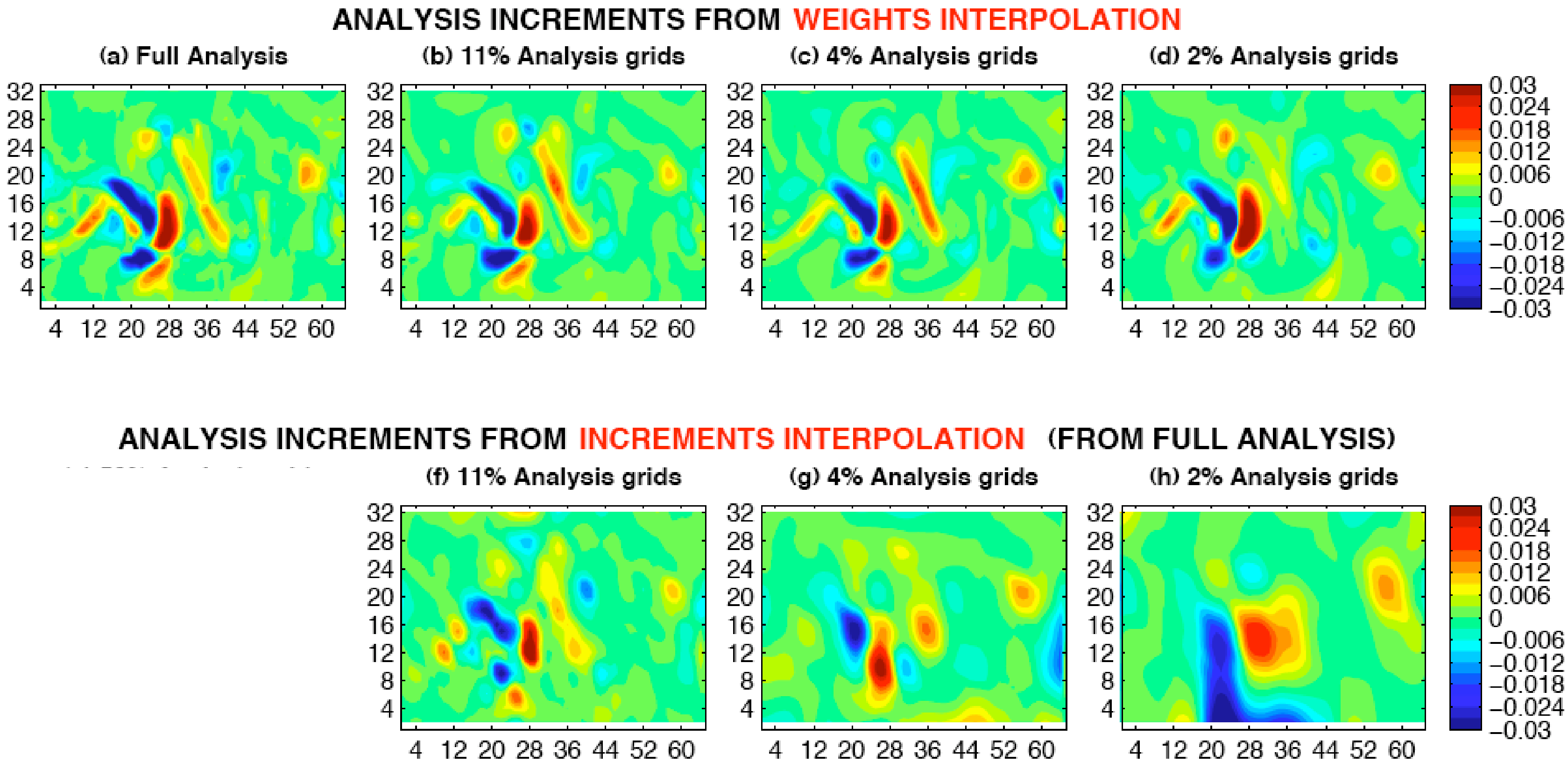
Yang et al (2008)

- In EnKF the analysis is a weighted average of the forecast ensemble
- We performed experiments with a QG model **interpolating** **weights** compared to **analysis** **increments**.
- Coarse grids of 11%, 4% and 2% interpolated analysis points.
- **Weight fields vary on large scales: they interpolate very well**



1/(3x3)=11% analysis grid

Weight interpolation versus **Increment** interpolation

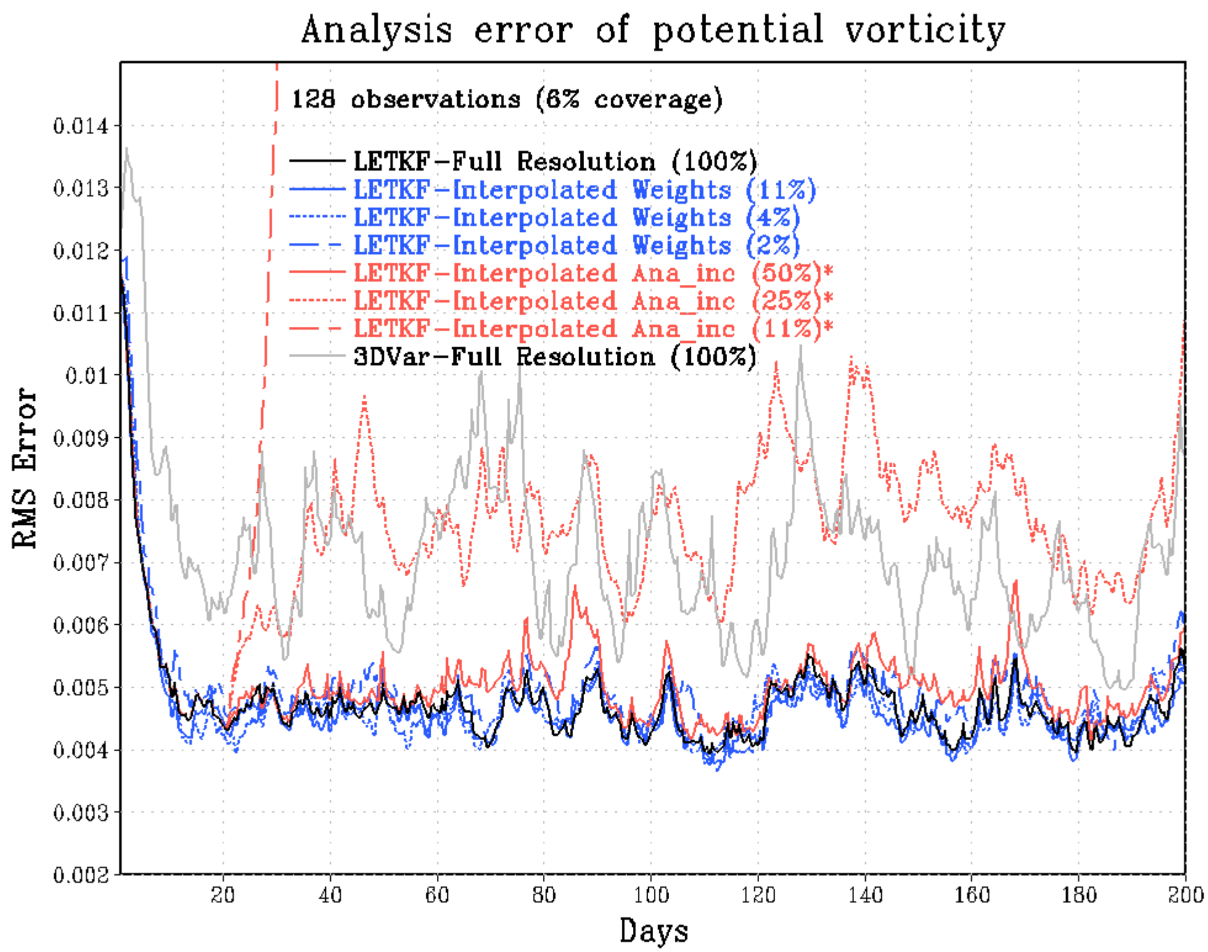


With increment interpolation, the analysis degrades quickly…

With weight interpolation, there is almost no degradation!

LETKF maintains balance and conservation properties

Impact of coarse analysis on accuracy



With increment interpolation, the analysis degrades

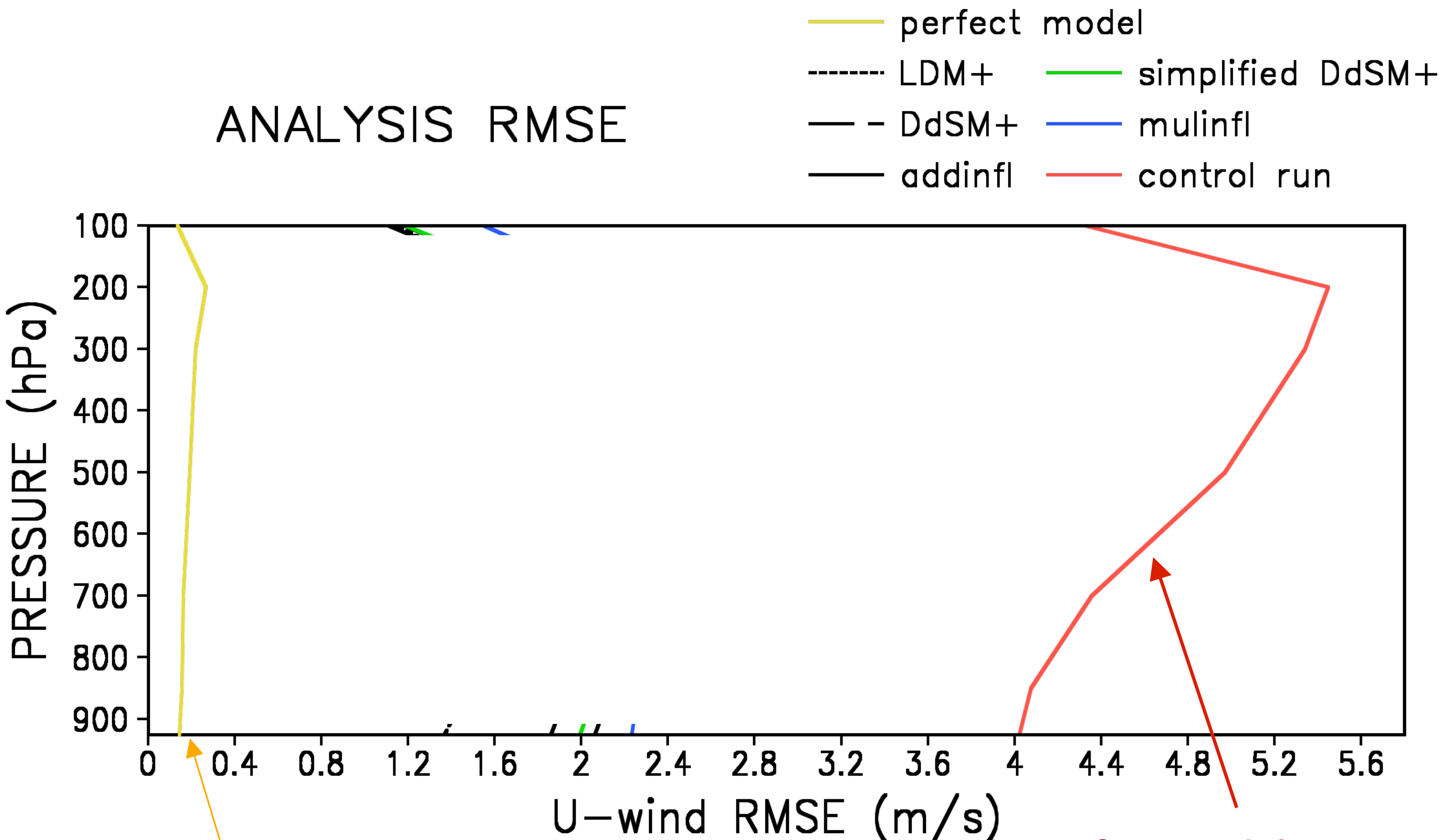
With weight interpolation, there is no degradation, the analysis is actually slightly better!

Model error: comparison of methods to correct model bias and inflation

Hong Li, Chris Danforth, Takemasa Miyoshi, and Eugenia Kalnay, MWR (2009)

Inspired by the work of Dick Dee, but with model errors estimated in model space, not in obs space

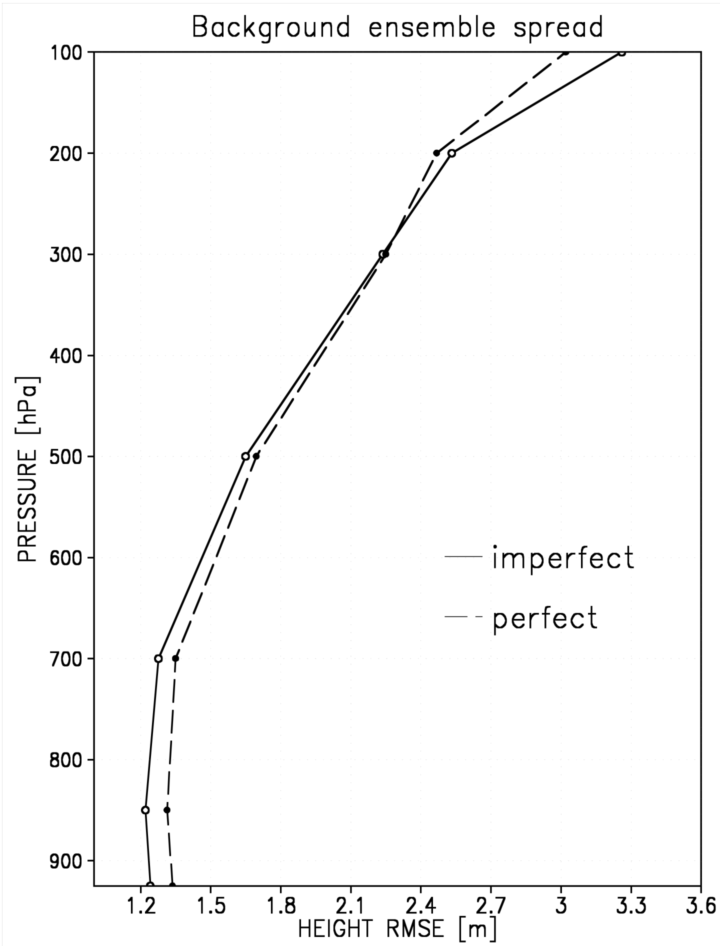
Model error: If we assume a perfect model in EnKF, we underestimate the analysis errors (Li, 2007)



perfect SPEEDY model

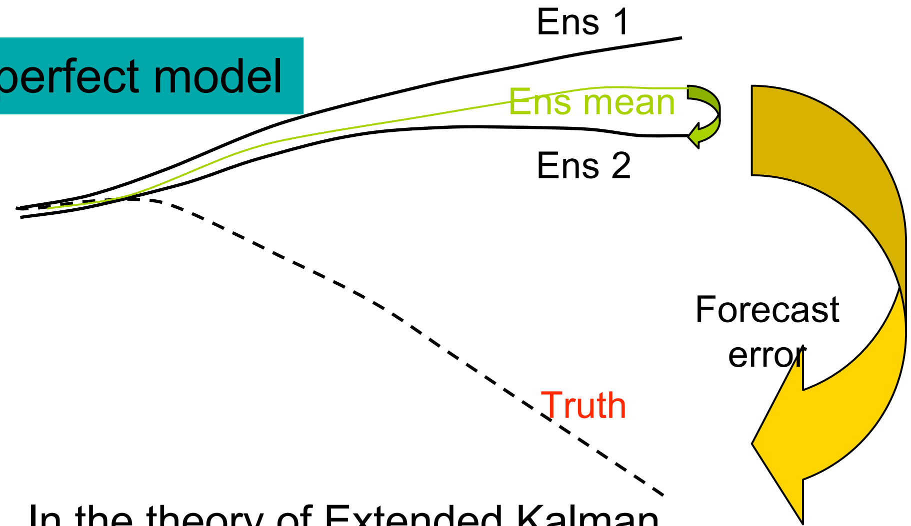
imperfect model (obs from NCEP- NCAR Reanalysis NNR)

— Why is EnKF vulnerable to model errors ?



The ensemble spread is 'blind' to model errors

Imperfect model



- In the theory of Extended Kalman filter, forecast error is represented by the growth of errors in IC and the model errors.

$$\mathbf{P}_i^f = \mathbf{M}_{\mathbf{x}_{i-1}^a} \mathbf{P}_{i-1}^a \mathbf{M}_{\mathbf{x}_{i-1}^a}{}^T + \mathbf{Q}$$

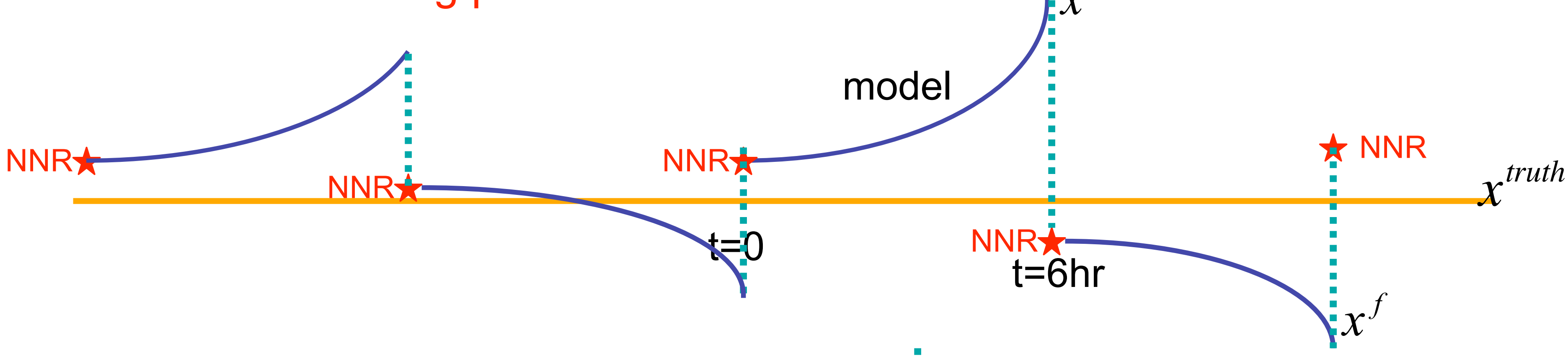
- However, in ensemble Kalman filter, error estimated by the ensemble spread can only represent the first type of errors.

$$\mathbf{P}_i^f \approx \frac{1}{k-1} \sum_{i=1}^{K} (x_i^f - \overline{x^f})(x_i^f - \overline{x^f})^T$$

Bias removal schemes (Low Dimensional Method)

2.3 Low-dim method (Danforth et al, 2007: Estimating and correcting global weather model error. *Mon. Wea. Rev, J. Atmos. Sci., 2007*)

- Generate a long time series of model forecast minus reanalysis x^e_{6hr} from the training period



We collect a large number of estimated errors and estimate from them bias, etc.

$$\varepsilon^f_{n+1} = \mathbf{x}^f_{n+1} - \mathbf{x}^t_{n+1} = \boxed{M(\mathbf{x}^a_n) - M(\mathbf{x}^t_n)} + \mathbf{b} + \sum_{l=1}^{L} \beta_{n,l}\mathbf{e}_l + \sum_{m=1}^{M} \gamma_{n,m}\mathbf{f}_m$$

Forecast error due to error in IC

Time-mean model bias

Diurnal model error

State dependent model error

Low-dimensional method

Include Bias, Diurnal and State-Dependent model errors:

$$\text{model error} = \mathbf{b} + \sum_{l=1}^{2} \beta_{n,l}\mathbf{e}_l + \sum_{m=1}^{10} \gamma_{n,m}\mathbf{f}_m$$

Having a large number of estimated errors allows to estimate the global model error beyond the bias

SPEEDY 6 hr model errors against NNR (diurnal cycle)

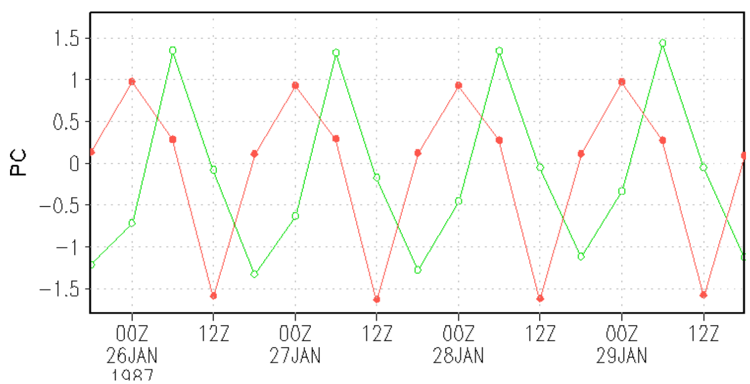
1987 Jan 1~ Feb 15

Error anomalies

$$x^{e\,'}_{6hr(i)} = x^{e}_{6hr} - \overline{x^{e}_{6hr}}$$

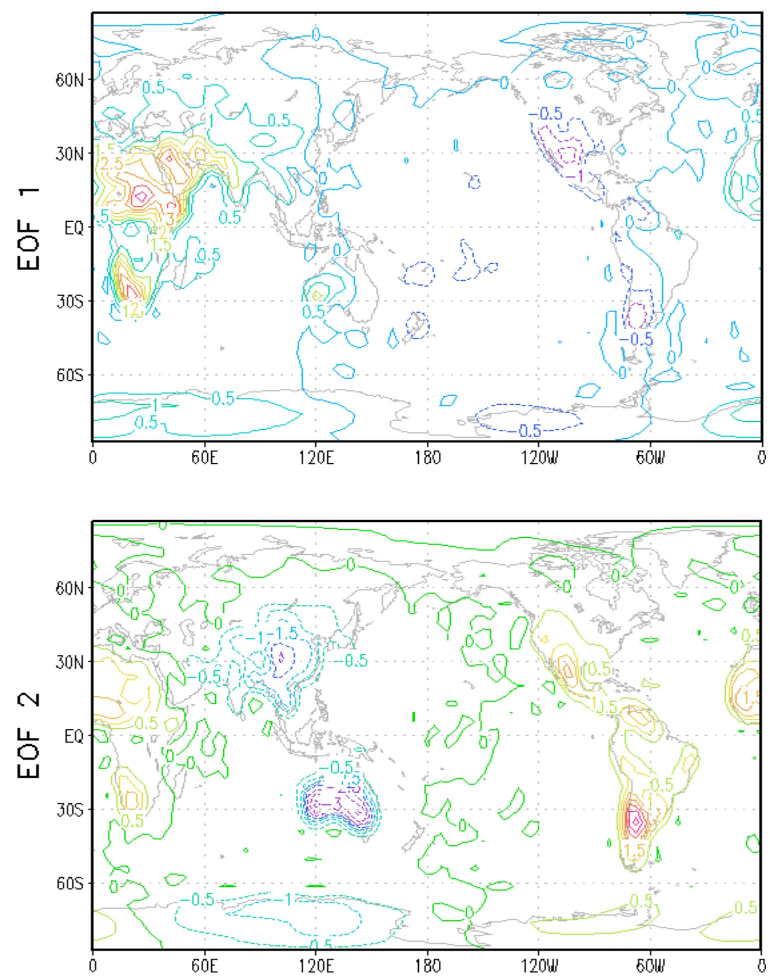
pc1

pc2

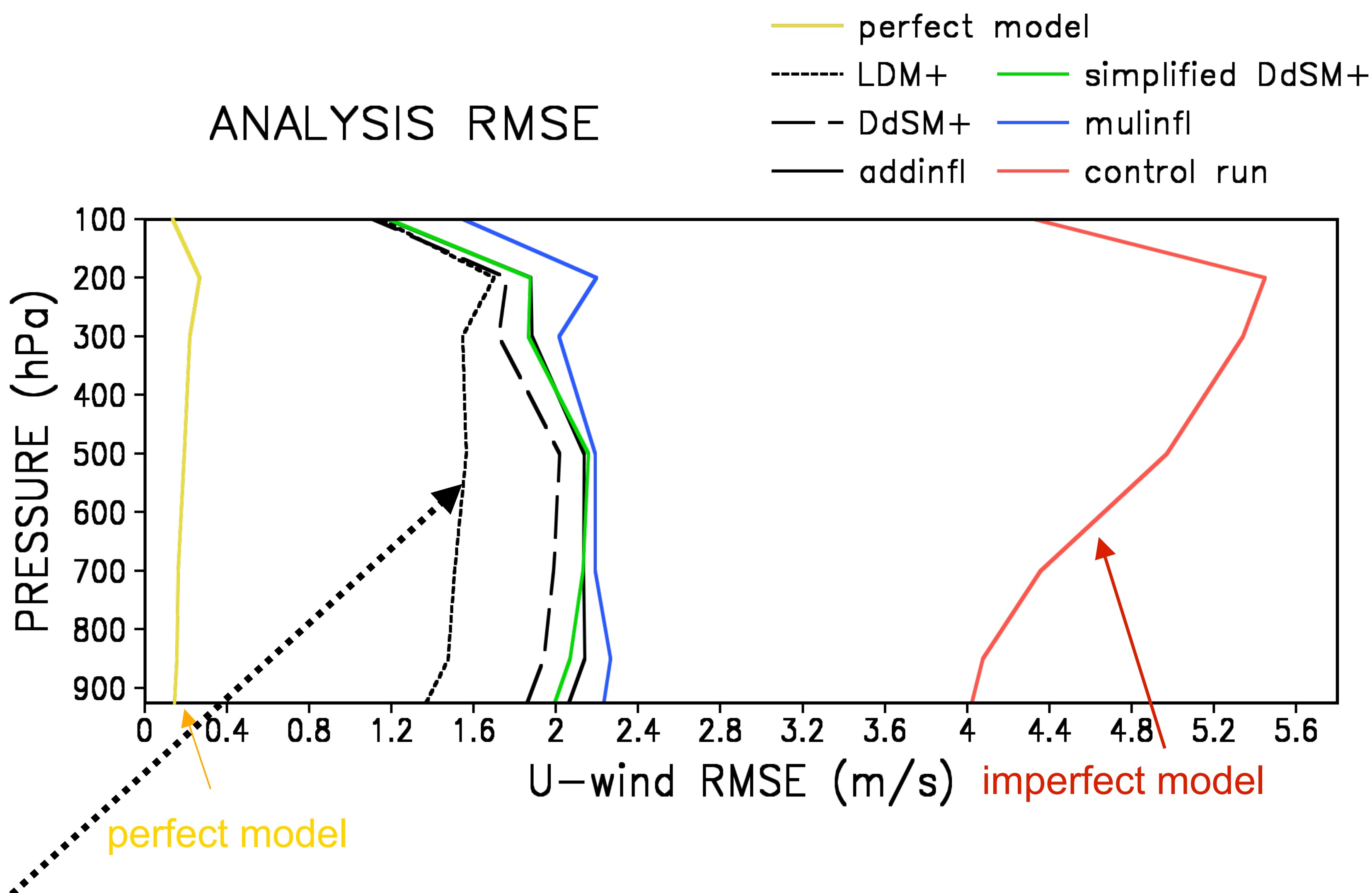


• For temperature at lower-levels, in addition to the time-independent bias, SPEEDY has diurnal cycle errors because it lacks diurnal radiation forcing

Leading EOFs for 925 mb TEMP



We compared several methods to handle bias and random model errors



Low Dimensional Method to correct the bias (Danforth et al, 2007) combined with additive inflation

Simultaneous estimation of EnKF inflation and obs errors in the presence of model errors

Hong Li, Miyoshi and Kalnay (QJ, 2009)

Inspired by Houtekamer et al. (2001) and Desroziers et al. (2005)

- Any data assimilation scheme requires accurate statistics for the observation and background errors (usually tuned or from gut feeling).
- EnKF needs inflation of the background error covariance: tuning is expensive
- Wang and Bishop (2003) and Miyoshi (2005) proposed a technique to estimate the covariance inflation parameter online. It works well if ob errors are accurate.
- We introduce a method to **simultaneously** estimate **ob errors** and **inflation**.

Diagnosis of observation error statistics

Houtekamer et al (2001) well known statistical relationship:

OMB*OMB $$< \mathbf{d}_{o-b}\mathbf{d}_{o-b}^{T} >= \mathbf{H}\mathbf{P}^{b}\mathbf{H}^{T} + \mathbf{R}$$

Desroziers et al, 2005, introduced two new statistical relationships:

OMA*OMB $$< \mathbf{d}_{o-a}\mathbf{d}_{o-b}^{T} >= \mathbf{R}$$

AMB*OMB $$< \mathbf{d}_{a-b}\mathbf{d}_{o-b}{}^{T} >= \mathbf{H}\mathbf{P}^{b}\mathbf{H}^{T}$$

These relationships are correct if the **R** and **B** statistics are correct and errors are uncorrelated!

With inflation: $\mathbf{H}\mathbf{P}^{b}\mathbf{H}^{T} \rightarrow \mathbf{H}\Delta\mathbf{P}^{b}\mathbf{H}^{T}$ with $\Delta > 1$

Diagnosis of observation error statistics

Transposing, we get "observations" of Δ and σ_o^2

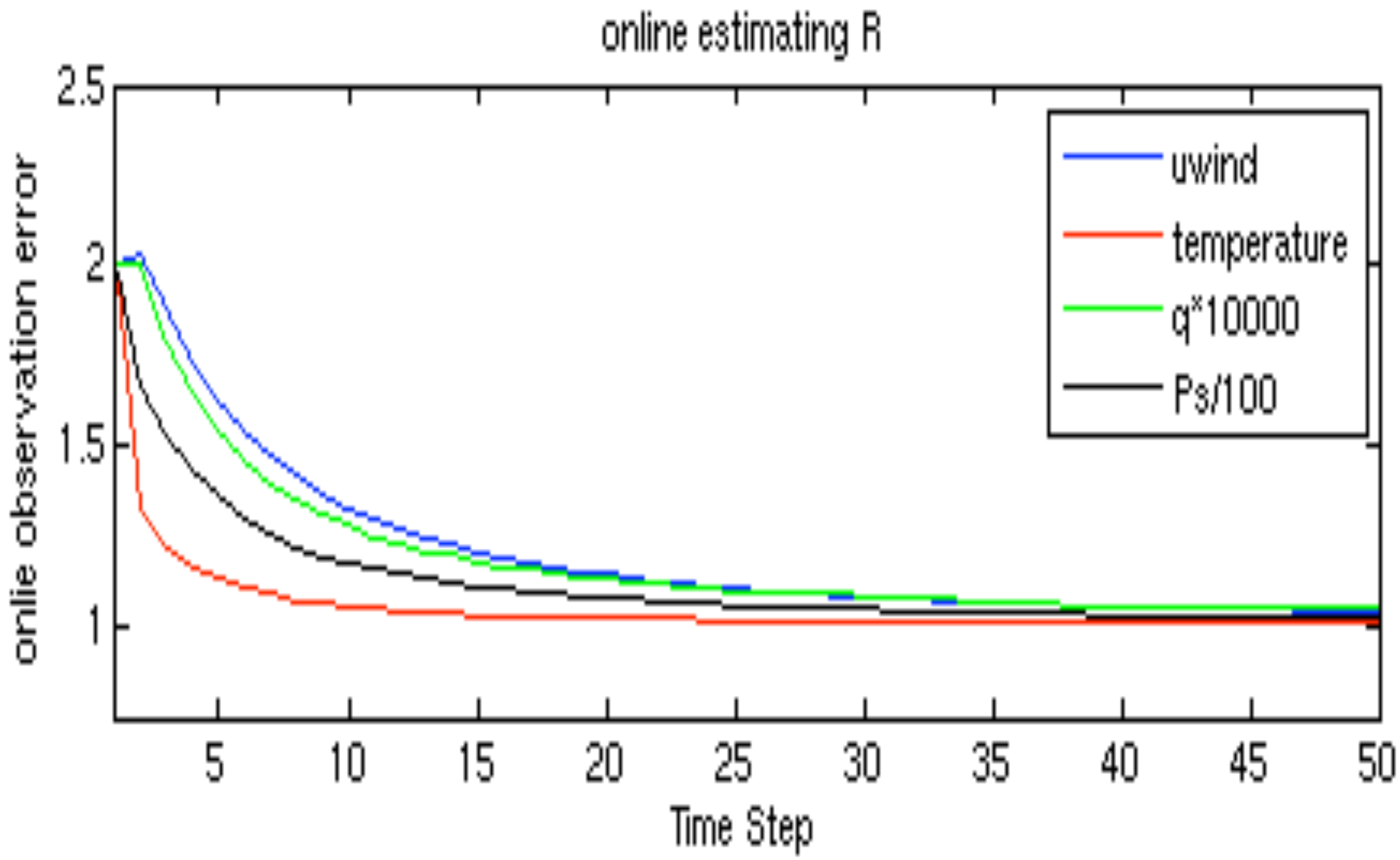
$$\Delta^o = \frac{(\mathbf{d}_{o-b}^T \mathbf{d}_{o-b}) - Tr(\mathbf{R})}{Tr(\mathbf{H}\mathbf{P}^b\mathbf{H}^\mathrm{T})}$$ OMB[2]

$$\Delta^o = \sum_{j=1}^{p} (y_j^a - y_j^b)(y_j^o - y_j^b) / Tr(\mathbf{H}\mathbf{P}^b\mathbf{H}^T)$$ AMB*OMB

$$(\tilde{\sigma}_o)^2 = \mathbf{d}_{o-a}^T \mathbf{d}_{o-b} / p = \sum_{j=1}^{p} (y_j^o - y_j^a)(y_j^o - y_j^b) / p$$ OMA*OMB

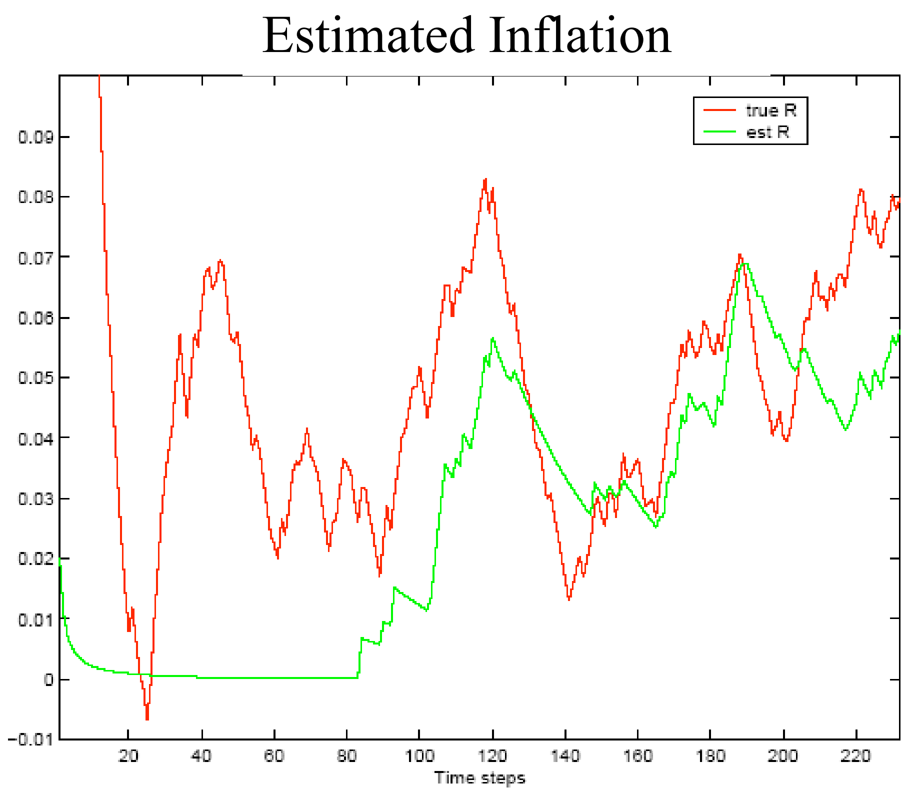
Here we use a simple KF to estimate both Δ and σ_o^2 online.

SPEEDY model: online estimated observational errors, each variable started with 2 not 1.



The original wrongly specified R quickly converges to the correct value of R (in about 5-10 days)

Estimation of the inflation



Using an initially wrong R and Δ but estimating them adaptively

Using a perfect R and estimating Δ adaptively

After **R** converges, the time dependent inflation factors are quite similar

Tests with LETKF with imperfect L40 model: added random errors to the model

Error amplitude (random)	A: true σ_o^2=1.0 (tuned) constant Δ		B: true σ_o^2=1.0 adaptive Δ		C: adaptive σ_o^2 adaptive Δ		
a	Δ	RMSE	Δ	RMSE	Δ	RMSE	σ_o^2
4	0.25	0.36	0.27	0.36	0.39	0.38	0.93
20	0.45	0.47	0.41	0.47	0.38	0.48	1.02
100	1.00	0.64	0.87	0.64	0.80	0.64	1.05

The method works quite well even with very large random errors!

Tests with LETKF with imperfect L40 model: added biases to the model

Error amplitude (bias)	A: true σ_o^2=1.0 (tuned) constant Δ		B: true σ_o^2=1.0 adaptive Δ		C: adaptive σ_o^2 adaptive Δ		
α	Δ	RMSE	Δ	RMSE	Δ	RMSE	σ_o^2
1	0.35	0.40	0.31	0.42	0.35	0.41	0.96
4	1.00	0.59	0.78	0.61	0.77	0.61	1.01
7	1.50	0.68	1.11	0.71	0.81	0.80	1.36

The method works well for low biases, but less well for large biases: Model bias needs to be accounted by a separate bias correction

Summary

- EnKF and 4D-Var give similar results in Canada and in JMA, except for model bias. (Buehner et al, Miyoshi et al)
- EnKF is better than GSI with half resolution model, 64 members. Computationally competitive (Whitaker)
- Many ideas to further improve EnKF were inspired in 4D-Var:
  - No-cost smoothing and "running in place"
  - A simple outer loop to deal with nonlinearities
  - Adjoint forecast sensitivity without adjoint model
  - Analysis sensitivity and exact cross-validation
  - Coarse resolution analysis without degradation
  - Correction of model bias combined with additive inflation gives the best results
  - Can estimate simultaneously optimal inflation and obs. errors

Miyoshi's LETKF code

Takemasa Miyoshi has a wonderful new LETKF code based on the work he did at JMA. He has made it available to all at "Google code Miyoshi LETKF".

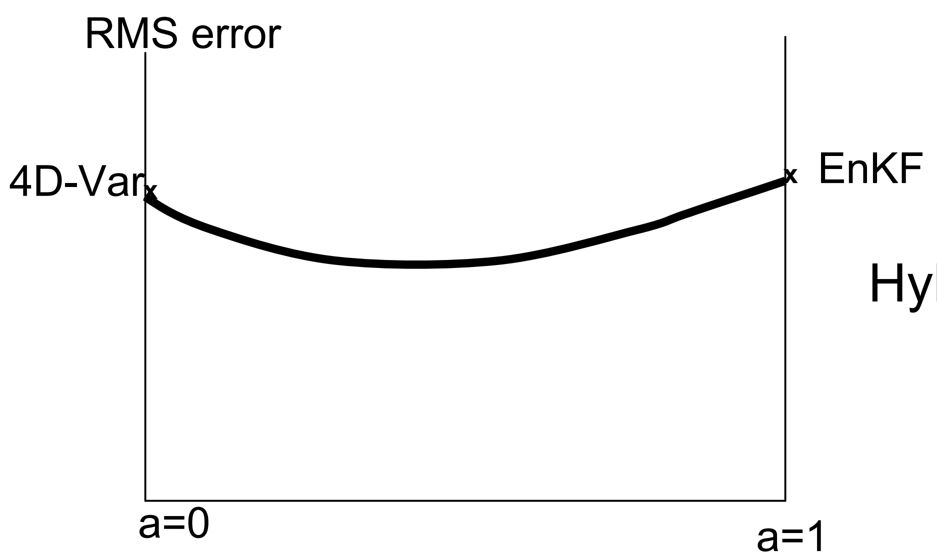
It is MPI (parallel) modular and very efficient, and the same code has been coupled to Lorenz (1996), SPEEDY and the Regional Ocean Modular System (ROMS) at high resolution.

Let me know if you need more information: you could run Lorenz-96 in a few minutes.

Thoughts on hybrid

Dale Barker suggested that a fast path for NCEP to the use of hybrid would be to make first a GSI-EnKF hybrid, and then replace GSI with 4D-Var. Seems a very sensible idea.

As shown by Buehner et al., hybrid Var and EnKF may be the most accurate approach ("sweet spot").



Hybrid=a*EnKF+(1-a)*4D-Var

Final thoughts

- EnKF is relatively new, but it has been shown to be better than 3D-Var and comparable to 4D-Var.
- It is simple to program and maintain: no LTM, no adjoint, no background error covariance, adapts to changes in observing systems.
- Ideally the tuning parameters (inflation and obs. errors, and localization) will be estimated adaptively. Miyoshi developed adaptive localization.
- Applications and properties of 4D-Var can be easily adapted to EnKF.